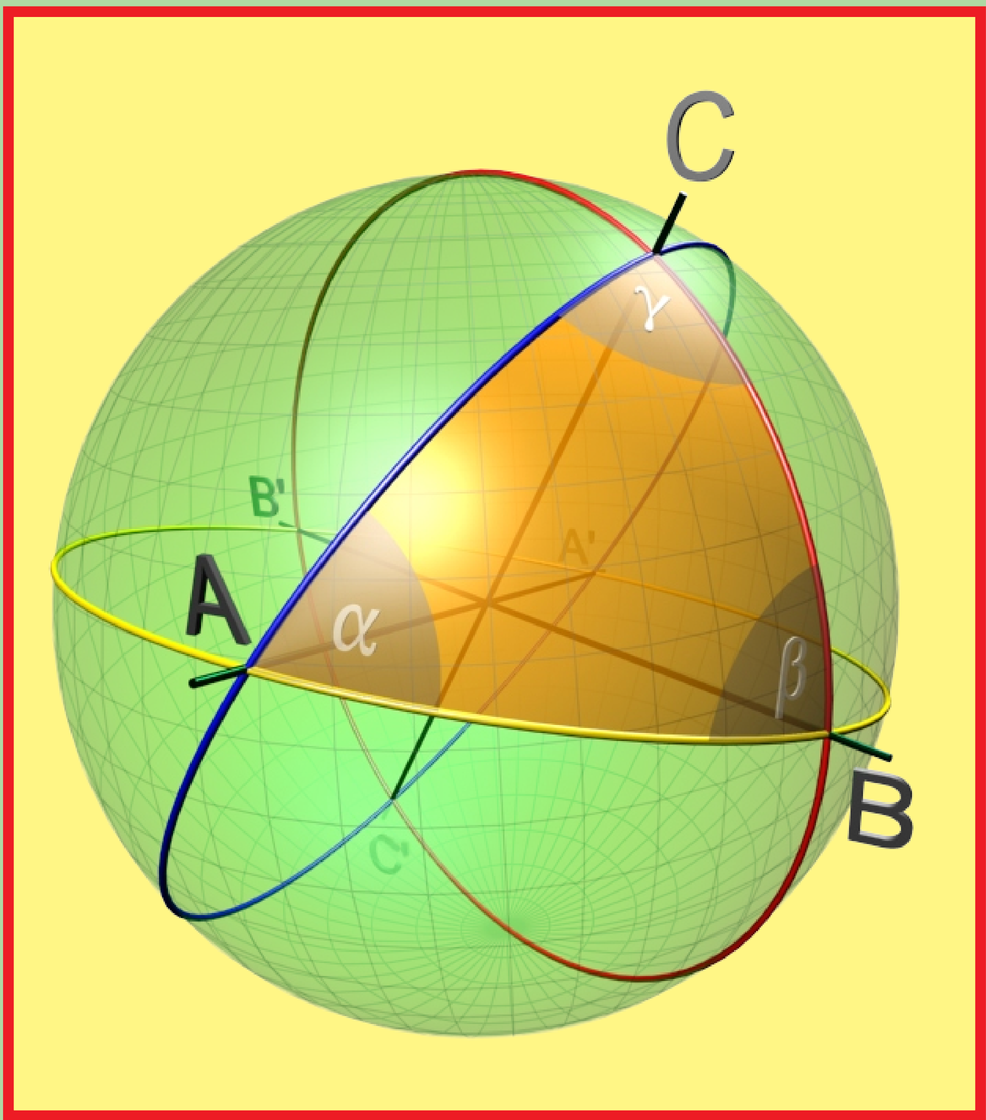
A Textbook of Spherical Trigonometry & Spherical Astronomy

S.K.D. DUBEY
D.S. PANDEY
B.D. DWIVEDI
AJIT KUMAR



A Textbook of Spherical Trigonometry & Spherical Astronomy

S.K.D. Dubey
D.S. Pandey
B.D. Dwivedi
Ajit Kumar

A Textbook of Spherical Trigonometry & Spherical Astronomy

S.K.D. Dubey
D.S. Pandey
B.D. Dwivedi
Ajit Kumar

Dominant
Publishers & Distributors Pvt Ltd
New Delhi, INDIA

Knowledge is Our Business

A TEXTBOOK OF SPHERICAL TRIGONOMETRY & SPHERICAL ASTRONOMY
By S.K.D. Dubey, D.S. Pandey, B.D. Dwivedi, Ajit Kumar

This edition published by Dominant Publishers And Distributors (P) Ltd
4378/4-B, Murarilal Street, Ansari Road, Daryaganj,
New Delhi-110002.

ISBN: 978-93-80642-10-9

Edition: 2022 (Revised)

Dominant
Publishers & Distributors Pvt Ltd

Registered Office: 4378/4-B, Murari Lal Street, Ansari Road,
Daryaganj, New Delhi - 110002.
Ph. +91-11-23281685, 41043100, Fax: +91-11-23270680

Production Office: "Dominant House", G - 316, Sector - 63, Noida,
National Capital Region - 201301.
Ph. 0120-4270027, 4273334

e-mail: dominantbooks@gmail.com
info@dominantbooks.com

w w w . d o m i n a n t b o o k s . c o m

CONTENTS

Chapter 1. Exploring Various Types of Coordinate Systems 1
— Ajit Kumar

Chapter 2. Analyzing the Horizontal Coordinate System 7
— Ashok Kumar

Chapter 3. Relationship between Horizontal and Hour Coordinate Systems 15
— Vipin Kumar

Chapter 4. A Review Study of Sun at the Zenith: Tropics 24
— Abhinav Saxena

Chapter 5. Exploring the Intriguing World of Coordinate Chirality 32
— Alok Kumar Gahlot

Chapter 6. Analyzing the Concept of Plane and Spherical Trigonometry 40
— Kamesh Kumar

Chapter 7. Geometry and Geography of Spherical Triangles 50
— Vijendra Singh Rawat

Chapter 8. Concept of Coordinates of the Azimuth Angle 58
— Rajiv Verma

Chapter 9. Exploring the Intricate World of Meteor Astronomy 64
— Puneet Sethi

Chapter 10. Eulerian Angle: Fundamental Concept of Mathematics 72
— Sunil Kumar Srivastava

Chapter 11. Coordinate Systems and Coordinate Transformations 78
— Abhinav Saxena

Chapter 12. Fundamental Concept of Astronomical Triangle 88
— Alok Kumar Gahlot

CHAPTER 1

EXPLORING VARIOUS TYPES OF COORDINATE SYSTEMS

Ajit Kumar, Associate Professor
Department of Computing Sciences & I.T., Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- ajit.chauhan79@gmail.com

ABSTRACT:

Coordinate systems are fundamental tools in mathematics and science, providing a framework for describing and understanding the spatial relationships of objects and phenomena. This chapter explores various types of coordinate systems, from the familiar Cartesian coordinates to spherical and polar coordinates, each tailored to address specific needs in different disciplines. We delve into their definitions, transformations, and practical applications, showcasing how they underpin a wide range of fields, from physics and engineering to astronomy and geography. As we journey through these systems, we highlight the power of coordinate systems as indispensable tools for problem-solving, modeling, and navigation in both two and three-dimensional spaces. By the chapter's end, readers will have gained a deeper appreciation for the elegance and versatility of coordinate systems we have embarked on a comprehensive exploration of coordinate systems, illuminating their significance in mathematics, science, and various applications. We began by introducing the foundational Cartesian coordinate system, which serves as the bedrock for countless mathematical and engineering endeavors. Transitioning to polar coordinates, we witnessed how a simple change in perspective can simplify complex problems, particularly in fields like physics and engineering.

KEYWORDS:

Axes, Cartesian, Celestial, Coordinate, Cylindrical, Equatorial.

INTRODUCTION

Astrometry is the branch of astronomy that measures the angles at which stars are separated from one another, under the assumption that the celestial sphere, of unit radius, is covered with stars. Astrometry's modern objectives include establishing basic reference systems, taking precise time measurements, accounting for precession and nutation, and figuring out the sizes and movements of the Galaxy[1], [2].Let's look at the fundamental concepts of geometry employed on the surface of a sphere as astrometry deals with arcs, angles, and triangles on the celestial sphere, whose characteristics differ from those of Euclidean geometry. These definitions will be used to explain the relationships between the components of a spherical triangle, the primary subject of study for spherical trigonometry, a mathematical approach that is used to handle observations and whose core ideas are provided[3], [4].

The branch of astronomy that deals with solving issues on the surface of the celestial sphere will be referred to as spherical astronomy. Finding the relationships between the various coordinate systems used in astronomy is one of the principal uses of the spherical astronomy formulas.The decision on which coordinate system to choose depends on the issue at hand, and the conversions between the systems enable measurements taken in one system to be transformed into another. You may achieve these changes by using either linear algebra or spherical trigonometry. Another intriguing use is the conversion of coordinate systems with Earth as their origin into coordinate systems with other planets, spacecraft, or the solar system's barycenter as their centers. This is particularly helpful for the study of the positions and motion of objects in the solar system.

Primary Definitions

In spherical astronomy, stars are seen as points on a sphere's surface with a single radius. A sphere is described as a two-dimensional surface where all points are equally far from a fixed point and is both finite and limitless. Spherical geometry, which is the area of mathematics that deals with curves that are arcs of large circles, is used on this surface. It is described below. Let's begin by outlining some of the fundamental ideas of spherical geometry that are relevant to planar two-dimensional surfaces and differ from those of Euclidian geometry in Figure 1.

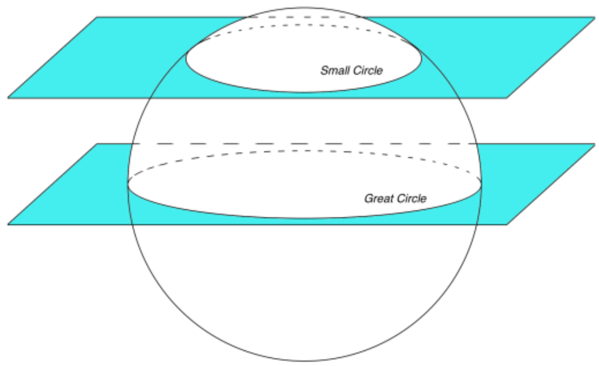


Figure 1: The intersection of a sphere with a plane is a circle.

Basic concepts

1. The intersection of a sphere with a plane is a circle.
2. Any plane that passes through the center of the sphere intercepts the sphere in a Great Circle.
3. Any circle, resulting from the intersection of the sphere with a plane that does not pass by the center, is called a small circle[5], [6].

We will be dealing mainly with great circles.

Theorem

The great circle that connects any two points A and B on the surface of a sphere defines the shortest path between these points (Figure 2).

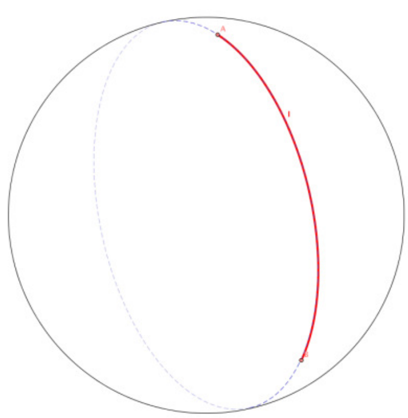


Figure 2: The shortest path l between two points A and B on a sphere is along an arc of great circle.

Proof: Given the conversion between Cartesian and spherical coordinates in Figure 3.

$$x = r\sin\theta\cos\phi \quad (1.1)$$
$$y = r\sin\theta\sin\phi \quad (1.2)$$
$$z = r\cos\theta \quad (1.3)$$

and the line element

$$d\boldsymbol{s} = dr\hat{\boldsymbol{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta\, d\phi\hat{\boldsymbol{\phi}}, \quad (1.4)$$

which we square to

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

On the surface of a unit sphere, $dr = 0$, and $r = 1$, so

$$ds^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

The length is

$$L = \int_A^B |ds| = \int_A^B \sqrt{d\theta^2 + \sin^2\theta\, d\phi^2} \quad (1.5)$$

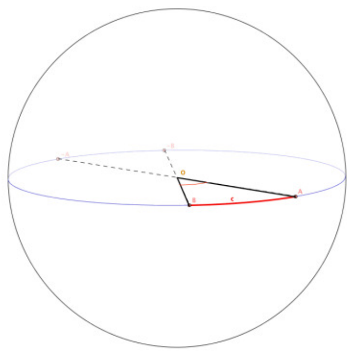


Figure 3: The angle AOˆB is equal to the angular size of the great circle arc c. On a sphere of unit radius, the angle AOˆB is identical to c.

The integrand can be written as in Figure 4:

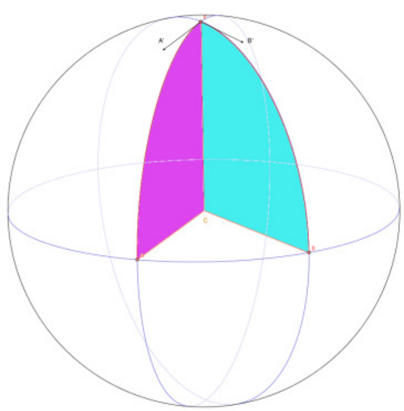


Figure 4: The spherical angle C is the dihedral angle between the planes that cross the sphere at the arcs AP and PB.

$$d\theta^2 + \sin^2\theta d\phi^2 = d\theta^2\left(1 + \sin^2\theta\,\frac{d\phi^2}{d\theta^2}\right)$$
$$= d\theta^2\left[1 + \sin^2\theta\left(\frac{d\phi}{d\theta}\right)^2\right] \quad (1.6)$$

Substituting Eq. (1.6) into Eq. (1.5)

$$L = \int_A^B \sqrt{1 + \left(\frac{d\phi}{d\theta}\right)^2 \sin^2\theta}\, d\theta \quad (1.7)$$

We can apply the chain rule to write

$$\frac{d\phi}{d\theta} = \frac{\partial\phi}{\partial\theta}\frac{\partial\theta}{\partial t} + \frac{\partial\phi}{\partial t} \quad (1.8)$$

And substituting Eq. (1.8) into Eq. (1.7)

$$L = \int_A^B \sqrt{1 + \phi'^2 \sin^2\theta}\, \theta' dt \quad (1.9)$$

This length is minimized if $d\phi' = 0$, i.e., if $\phi = \text{const}$. This means it must lie along a meridian (defined as great circles of constant longitude).

DISCUSSION

The angle, in radians, subtended at the center of a sphere of unit radius is equal to the arc of a great circle joining two points on the surface. This is closely related to how a circle's length is defined. The angle AOB in the picture is where the length of the route, c, is equal to R. If R = 1, and c have the same units as each other. The diameter of the sphere, perpendicular to the great circle, and the spherical surface meet at the poles (P, P0) of a great circle. The poles are antipodes (diametrically opposing positions), which means that they are 180 degrees apart[7], [8].

Circular Angle

The spherical angle (dihedral angle), which is created when two great circle arcs collide, is the angle between their planes. The angle between the tangents (PA0, PB0) to both of the great circle arcs at their intersection point is another way to describe the spherical angle. Elements: The arcs of the great circle make up the sides of the spherical angle, and their intersection is where we get the vertex. The sides of the figure are PA and PB, and P serves as the vertex.

Coordination Systems for Basics

Observer's overhead point is the zenith.

An antipode is a point on a sphere's surface that is directly opposite another point.

Nadir is the opposite of zenith.

Meridian: A North-South line that crosses the zenith.

1. Diurnal motion is the celestial sphere's daily east-west revolution. A star's diurnal journey circles the celestial pole in a relatively tiny circle.
2. The culmination of a star's daily motion are its meridian passes. Both higher and lower culminations exist.

3. Two essential big circles identified in the celestial sphere are the celestial equator and the ecliptic. The Earth's equator is projected onto the celestial sphere as the celestial equator. The Earth's orbit in the celestial sphere is represented by the ecliptic. From our perspective, it is the route left behind by the Sun's yearly angular movement in the sky.

Equinox: also known as the "equal night," is a Latin word. When the Sun is near the celestial equator, day and night last the same amount of time.

1. Latin for "sun stop" is "solstice." the moment the Sun reverses its yearly course from north to south, designating either the longest or shortest night of the year.
2. Vernal Point is one of the points where the celestial equator and the ecliptic converge. also known as the vernal equinox or the Aries first point. The Autumnal Point, also known as the Autumnal Equinox or the First Point of Libra, is the antipode of the Vernal Point.

Sidereal: relating to the stars (from the Latin starus). The time it takes to return to the same location with regard to far-off stars is referred to as a sidereal period[9], [10].

Synodic: In relation to alignment with another celestial body, usually the Sun (from the Greek sunodos, gathering or meeting). The time it takes to return to the same location with regard to the Sun is often referred to as a synodic period. Another point in the sky is often used as a reference for the Sun (either the meridian for the synodic day or the vernal point for the synodic year).

CONCLUSION

In our investigation of spherical coordinates, we showed how useful they are for describing things on a sphere's surface and how crucial they are to astronomy and geography. By navigating the complexities of cylindrical coordinates, we were able to demonstrate how these systems simplify certain three-dimensional issues, such as those that arise in fluid dynamics.

We stressed the flexibility and adaptability of coordinate systems throughout our trip to meet the requirements of many disciplines. We learned that they are useful tools for problem-solving, modeling, and navigating in the actual world, not simply abstract mathematical abstractions. As we draw to a close, we want readers to acknowledge the ongoing importance of coordinate systems. These systems serve as the universal language of spatial connections, from the computer visuals that amuse us to the global positioning systems that direct us. Coordinate systems are the weave and weft that join the threads of knowledge and discovery in the vast fabric of mathematics and science. We hope that this chapter has increased your understanding of the beauty and strength of coordinate systems, and we cordially welcome you to continue exploring the wide range of applications that rely on their accuracy and adaptability.

REFERENCES:

[1] R. J. van Arkel and J. R. T. Jeffers, “In vitro hip testing in the International Society of Biomechanics coordinate system,” *J. Biomech.*, 2016, doi: 10.1016/j.jbiomech.2016.10.036.

[2] T. E. Hudson and M. S. Landy, “Sinusoidal error perturbation reveals multiple coordinate systems for sensorymotor adaptation,” *Vision Res.*, 2016, doi: 10.1016/j.visres.2015.12.005.

[3] M. Szabova and F. Duchon, “Survey Of GNSS Coordinates Systems,” *Eur. Sci. Journal, ESJ*, 2016, doi: 10.19044/esj.2016.v12n24p33.

[4] B. G. Choi, Y. W. Na, and J. W. Jung, “Small scale map projection and coordinate system improvement in consideration of usability and compatibility,” *J. Korean Soc. Surv. Geod. Photogramm. Cartogr.*, 2016, doi: 10.7848/ksgpc.2016.34.2.171.

[5] H. Mecheri, X. Robert-Lachaine, C. Larue, and A. Plamondon, “Evaluation of Eight Methods for Aligning Orientation of Two Coordinate Systems,” *J. Biomech. Eng.*, 2016, doi: 10.1115/1.4033719.

[6] J. Sladek and A. Gaska, “Modelling of the coordinate measuring systems accuracy,” *J. Mach. Eng.*, 2016.

[7] I. Stiperski and M. W. Rotach, “On the Measurement of Turbulence Over Complex Mountainous Terrain,” *Boundary-Layer Meteorol.*, 2016, doi: 10.1007/s10546-015-0103-z.

[8] J. Chen, F. Liu, F. Zhao, G. Zhu, and S. Ding, “A sc-vivaldi network coordinate system based method for IP geolocation,” *J. Internet Technol.*, 2016, doi: 10.6138/JIT.2016.17.1.20151109a.

[9] T. B. Britton *et al.*, “Tutorial: Crystal orientations and EBSD - Or which way is up?,” *Mater. Charact.*, 2016, doi: 10.1016/j.matchar.2016.04.008.

[10] S. Calinon, “Stochastic learning and control in multiple coordinate systems,” *Int. Work. Human-Friendly Robot.*, 2016.

CHAPTER 2

ANALYZING THE HORIZONTAL COORDINATE SYSTEM

Ashok Kumar, Assistant Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- ashoksiddhu111@gmail.com

ABSTRACT:

The Horizontal Coordinate System is a fundamental celestial coordinate system used by astronomers and stargazers to locate and observe celestial objects in the night sky. In this chapter, we explore the intricacies of this system, which provides a user-friendly way to describe the positions of stars, planets, and other celestial bodies as they appear from a specific observer's location on Earth. We delve into the key concepts of azimuth and altitude, which define an object's position in the sky relative to the observer's local horizon. Additionally, we discuss the practical applications of the Horizontal Coordinate System, including its role in celestial navigation, telescope alignment, and stargazing. By the chapter's end, readers will have a solid understanding of how this coordinate system simplifies the experience of exploring the wonders of the cosmos from our own vantage point on Earth. we have embarked on a journey through the Horizontal Coordinate System, a vital tool in the arsenal of astronomers and stargazers alike. We began by introducing the key concepts of azimuth and altitude, which are the cornerstones of this coordinate system. Azimuth, measured in degrees clockwise from the north, tells us the direction of a celestial object from our viewpoint, while altitude, measured in degrees above the horizon, reveals its height in the sky. These two values, combined, provide an intuitive way to describe where an object appears in our night sky.

KEYWORDS:

Geographical, Geographic, Horizontal, Latitude, Longitude, Spherical.

INTRODUCTION

Azimuth A (often measured from the southern point, westward 1) and altitude h (measured from the horizon to the zenith) are the coordinates. In place of altitude, the zenital distance, z = 90 h, is often utilized. Almucantars are defined as stars at the same height. A little circle perpendicular to the horizon is known as an almucantar [1], [2].

System of Equatorial Coordinates

Celestial equator is the fundamental plane (see Figure 1). Right ascension and declination 6 are the coordinates. As a perpendicular from the celestial equator to the star, the declination 6 is calculated. From the vernal point, the right ascension is measured eastward. The celestial equator and the ecliptic, which is the path left in the sky by the Sun's yearly journey, cross at the vernal point [3], [4].

1. Hour Celestial equator is the fundamental plane of the coordinate system.
2. Hour angle H (measured from meridian) and declination 6 (the same as in equatorial coordinates) are the coordinates.

An hour circle, which is a North-South great circle, is defined by stars with the same hour angle. At the same moment, the stars in the same hour circle peak. The hour angle is a unit of time that ranges from 0 to 24 hours and corresponds to the interval since the last upper culmination. The yearly motion of the Sun in the sky, which is a projection of the Earth's orbital motion, defines the ecliptic coordinate system's fundamental plane (Figure 2). Ecliptic longitude (as measured from the vernal point) and ecliptic latitude (as measured from the ecliptic) are the coordinates. The axial tilt of the Earth with respect to its orbit, s = 232702600, causes the Ecliptic to be inclined to the celestial equator. Fundamental plane in the cosmic coordinate system: the galactic plane. Galactic longitude, calculated from the galactic center, and galactic latitude, calculated from the galactic plane [5], [6].

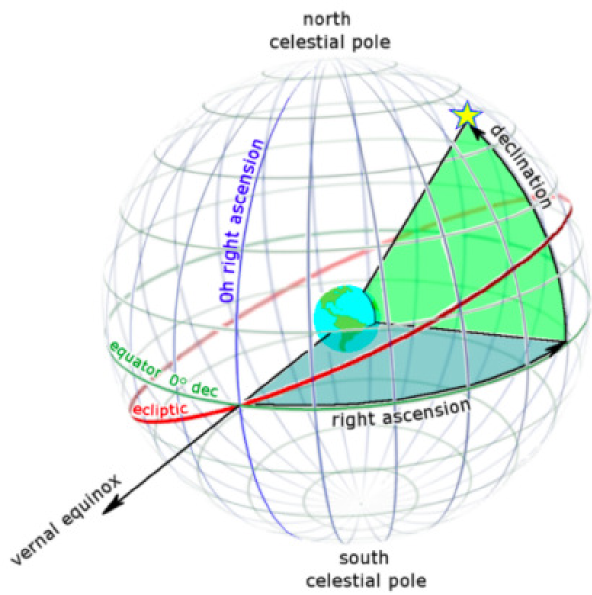


Figure 1: Equatorial coordinate system.

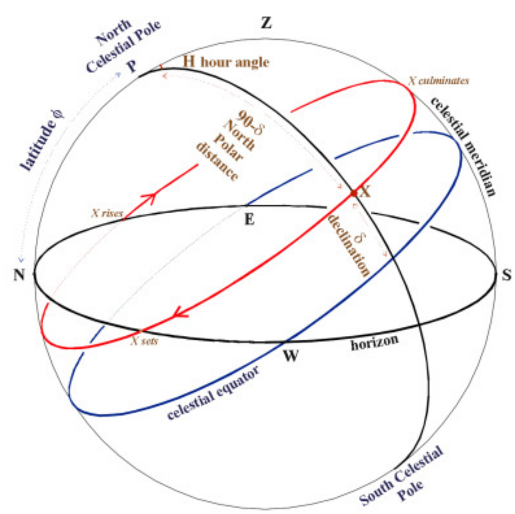


Figure 2: Hour coordinate system. The hour angle is measured from the meridian to the hour circle of the star.

Spherical triangle

The shape made up of three points linked by pairs and three spots on the surface of a sphere where arcs of a great circle intersect is known as a spherical triangle. Every side and angle of an Eulerian spherical triangle is less than 180 degrees [7], [8].

Corollary 1: A plane that does not traverse the sphere's center is defined by three locations that are not a part of the same great circle.

Corollary 2: The three points may be placed on the sphere such that they are always in the same hemisphere, which is the second corollary. Therefore, no angle in the spherical triangle may have a length greater than 180.

Corollary 3: Only large circle arcs may be found in a spherical triangle. Arcs made from tiny circles cannot be used to create it.

1. The spherical triangle is made up of six parts: 3 sides, as opposed to the angles, are often denoted by lowercase letters (BC = a, CA = b, AB = c), whereas 3 angles are typically denoted by capital letters (ABC).
2. The spherical triangle's vertices are the same as the vertices of the spherical angles. The arcs of the three great circles make up the sides (AB, BC, and CA).
3. The dihedral angles are used to calculate the angles (A, B, and C).

Properties

The following properties are valid for Eulerian triangles, i.e., those for which each side or angle does not exceed 180◦.

1. The sum of the three sides of a spherical triangle is between 0◦ and 360◦ (2⇡).

$0^\circ < a + b + c < 360^\circ$

2. The sum of the three angles of a spherical triangle is greater than 180◦ (⇡) and smaller than 540◦ (3⇡)

$180^\circ < A + B + C < 540^\circ$

3. One side is greater than the difference of the two others and smaller than the sum of two other sides.

$|b - c| < a < b + c$

4. When two sides are equal, the two opposite angles are also equal and vice-versa.

a = b () A = B

5. The order in which the values of the sides of a spherical triangle are distributed is the same in which the angles are distributed

a < b < c () A < B < C

Spherical Trigonometry

The branch of mathematics known as spherical trigonometry examines the connections between the six componentsthree angles and three sidesof a spherical triangle [9], [10].

The basic rule of cosines

Let u, and w be unit vectors from the sphere's center to the triangle's corners as we consider a spherical triangle.

Align u with the z-axis while maintaining generality, and make u lie on the x-z plane. Angle is calculated from the pole in spherical coordinates. In spherical coordinates, the vector is thus:

$$\hat{v} = (1, a, 0) \tag{1.15}$$

For the vector $\hat{w}$, the have $\theta = b$ and $\phi = C$, so its coordinates are

$$\hat{w} = (1, b, C) \tag{1.16}$$

In Cartesian coordinates (Eq. (1.1), Eq. (1.2), and Eq. (1.3)) the vectors $\hat{v}$ are $\hat{w}$ are written

$$\hat{v} = (\sin a, 0, \cos a) \tag{1.17}$$

$$\hat{w} = (\sin b \cos C, \sin b \sin C, \cos b) \tag{1.18}$$

Because c is an arc of great circle and the sphere has unit radius, c is the angle between $\hat{v}$ and $\hat{w}$, as this is also the angle these vectors subtend from the center of the sphere (i.e., c is the angle $B\hat{O}A$). The dot product between $\hat{v}$ and $\hat{w}$ is therefore

$$\hat{v} \cdot \hat{w} = |\hat{v}|\,|\hat{w}|\,\cos c$$

And because $\hat{v}$ and $\hat{w}$ are unit vectors, this reduces to

$$\cos c = \hat{v} \cdot \hat{w}.$$

The dot product is

$$(\sin a, 0, \cos a) \cdot (\sin b \cos C, \sin b \sin C, \cos b) = \cos b \cos a + \sin a \sin b \cos C$$

Leading to what is known as the fundamental law of cosines

$$\boxed{\cos c = \cos b \cos a + \sin a \sin b \cos C} \tag{1.19}$$

If we cyclically permutate the elements of Eq. (1.19) we obtain the following group

$$\boxed{\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C \end{aligned}} \tag{1.20}$$

Solved Problem

Problem: LA is at coordinates (34.0522, -118.2437), while New York is at (40.7128, -74.0060).

Solution: Let's build the spherical triangle based on the separations. The cities serve as vertices, and the third vertex is the North Pole. The cities' latitudes are calculated from the equator outward.

The co-latitudes, 90° 11 and 90° 12, are therefore two of the triangle's sides. We are looking for the distance between the cities, which corresponds to the arc c.

The deference of the cities' longitudes, measured from point (0,0), where the Greenwich meridian meets the equator, and the locations L2 and L1 at the equator, is the angle produced by the arcs that meet at the pole. We can then solve the triangle.

cos c = cos b cos a + sin b sin a cos C

with b = 90 – l1, a = 90 – l2, and C = L1 – L2. We find c = 35.3950◦. The distance, given the Earth's radius R = 6 371 km, is R c = 3936 km.

Dual triangles

The dual triangle of a spherical triangle ABC, with vertices ABC, is the triangle A0B0C0, with vertices the poles of the great circles that constitute ABC. Green denotes the original triangle ABC. The dashed lines represent the perpendicular lines to the segment vertices. These lines converge at the points A0B0C0 to form the dual triangle, which is shown in pink. All red lines are 90-degree arcs.

The twin triangle's property states that its angles are extensions of the original triangle's sides.

a0 = ⇡ – A (1.22)

Proof: Given A0B0C0 the poles of abc, a0 = B0C0 is on the equator of A. We can prolong the arcs AB and AC until they reach the equator, defining the points P and Q. The angular size of PQ is equal to A, as the angle PAˆQ is the same as BAˆC.

Because Q is on the equator of B0, the arc B0Q = ⇡/2. Similarly, because P is on the equator of C0, the arc C0P = ⇡/2.

By geometrical construction,

$$\begin{aligned} B'C' &= B'P + PQ + QC' \\ &= B'Q + QC' \\ &= \frac{\pi}{2} + QC' \end{aligned} \qquad (1.23)$$

Also by geometrical construction,

$$\begin{aligned} QC' &= PC' - PQ \\ &= \frac{\pi}{2} - A \end{aligned} \qquad (1.24)$$

And thus

$$B'C' = \pi - PQ \qquad (1.25)$$

And because $B'C' = a'$ and $PQ = A$, we arrive at the proof

$$\boxed{a' = \pi - A.} \qquad (1.26)$$

Cosine law for angles

Due to the fact that it connects a side to an angle, it is of utmost significance. We may utilize this connection to swap sides by angles if we establish a theorem for the sides of a spherical triangle. For instance, the link established by Equation (1.26) and Equation (1.19), when combined, result in the cosine law for angles.

$$\boxed{\begin{array}{l}\cos A = -\cos B \cos C + \sin B \sin C \cos a \\ \cos B = -\cos C \cos A + \sin C \sin A \cos b \\ \cos C = -\cos A \cos B + \sin A \sin B \cos c\end{array}}$$

Law of sines

Starting from sin2 A = 1 – cos2 A, and substituting cos A from the law of cosines.

$$\begin{aligned}\sin^2 A &= 1 - \left(\frac{\cos a - \cos b \cos c}{\sin b \sin c}\right)^2 \\ &= \frac{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}{\sin^2 b \sin^2 c} \qquad (1.28)\end{aligned}$$

We take the square root and divide by $\sin a$

$$\frac{\sin A}{\sin a} = \frac{\left[1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c\right]^{1/2}}{\sin a \sin b \sin c} \qquad (1.29)$$

The RHS does not depend on combinations of a,b,c, thus it must be the same for b and c

$$\boxed{\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}} \qquad (1.30)$$

Arc and angle formula

From the triangles, a last useful trigonometric connection is formed. It is a /2 arc, the arc AD. There are two methods for determining d's length. ABD from the triangle,

$$\begin{aligned}\cos d &= \cos\left(\frac{\pi}{2}\right)\cos c + \sin\left(\frac{\pi}{2}\right)\sin c \cos A \\ &= \sin c \cos A \qquad (1.31)\end{aligned}$$

and from the triangle CBD

$$\begin{aligned}\cos d &= \cos\left(\frac{\pi}{2} - b\right)\cos a + \sin\left(\frac{\pi}{2} - b\right)\sin a \cos(\pi - C) \\ &= \sin b \cos a - \cos b \sin a \cos C \qquad (1.32)\end{aligned}$$

Equating Eq. (1.131) and Eq. (6.181)

$$\boxed{\sin c \cos A = \sin b \cos a - \cos b \sin a \cos C} \qquad (1.33)$$

If we use the dual triangle relations, $a' = \pi - A$, we have the same relation with inverted angles and sides

$$\boxed{\sin C \cos a = \sin B \cos A + \cos B \sin A \cos c} \qquad (1.34)$$

Applying the equations

1. The spherical triangle's known components determine the spherical trigonometry formulae that must be used to solve difficulties. Consequently, there are four stages that may be used to solve a problem:

2. Build the spherical triangle using the problem data as a starting point. Use the poles and the fundamental circle of the coordinate system used in the problem as reference points.
3. List the components we are aware of and those we wish to learn more about.
4. Decide which formulae will best solve the issue. When there are several ways to solve a problem, we must choose the solution that is the most straightforward—that is, requires the fewest computations. We categorize the formulae according to the components we wish to link in order to make the decision easier.
5. After finding the answer, we must confirm the outcomes. Because the value of the element must be within the range of 0 and 180, it is precisely known whether it is supplied by a cosine, tangent, or cotangent. However, the sine will produce two new arcs or two additional angles that will solve the issue.

CONCLUSION

We looked at how the Horizontal Coordinate System, which adjusts to any observer's position on Earth, makes stargazing and celestial navigation accessible to everyone. This approach makes it easier to locate celestial objects and determine their routes over the firmament, whether you're a novice astronomer putting up your telescope or a navigator getting your bearings by the stars. The use of the Horizontal Coordinate System for telescope alignment, which enables astronomers to precisely target celestial objects, and its function in celestial navigation, which has guided explorers and sailors across the seas for centuries, were other practical applications that we delved into. We ask readers to test their newfound understanding of the Horizontal Coordinate System by going outdoors on a clear night once we have finished our research. Look up, recognize the stars, and join the countless generations of awestruck astrologers who have done the same. In order to explore the vastness of the night sky and to establish a closer relationship with the universe that extends above us, the Horizontal Coordinate System acts as a link between Earth and the cosmos.

REFERENCES:

[1] E. Hofmann, R. Fimmers, M. Schmid, U. Hirschfelder, A. Detterbeck, and K. Hertrich, “Landmarks of the Frankfort horizontal plane□: Reliability in a three-dimensional Cartesian coordinate system.,” *J. Orofac. Orthop.*, 2016.

[2] A. G. Patterson *et al.*, “Quorum Sensing Controls Adaptive Immunity through the Regulation of Multiple CRISPR-Cas Systems,” *Mol. Cell*, 2016, doi: 10.1016/j.molcel.2016.11.012.

[3] A. E. T. Yang and M. J. Z. Hartmann, “Whisking kinematics enables object localization in head-centered coordinates based on tactile information from a single vibrissa,” *Front. Behav. Neurosci.*, 2016, doi: 10.3389/fnbeh.2016.00145.

[4] Y. Wei *et al.*, “Design and analysis of the optical structure for K mirror in astronomical telescopes,” *Zhongguo Jiguang/Chinese J. Lasers*, 2016, doi: 10.3788/CJL201643.0316002.

[5] M. Berber and W. Wright, “Online kinematic GNSS data processing for small hydrographic surveys,” *Ocean Eng.*, 2016, doi: 10.1016/j.oceaneng.2015.10.001.

[6] G. Svenson, P. Flisberg, and M. Rönnqvist, “Using Analytics in the Implementation of Vertical and Horizontal Curvature in Route Calculation,” *IEEE Trans. Intell. Transp. Syst.*, 2016, doi: 10.1109/TITS.2015.2503424.

[7] G. Casula, “Geodynamics of the Calabrian Arc area (Italy) inferred from a dense GNSS network observations,” *Geod. Geodyn.*, 2016, doi: 10.1016/j.geog.2016.01.001.

[8] H. M. S. Bahaidarah, “Entropy generation during fluid flow in sharp edge wavy channels with horizontal pitch,” *Adv. Mech. Eng.*, 2016, doi: 10.1177/1687814016660929.

[9] T. Aizawa and H. L. Tanaka, “Axisymmetric structure of the long lasting summer Arctic cyclones,” *Polar Sci.*, 2016, doi: 10.1016/j.polar.2016.02.002.

[10] J. Gašinec and S. Gašincová, “Landslide deformation analysis based on robust m-estimations,” *Inz. Miner.*, 2016.

CHAPTER 3

RELATIONSHIP BETWEEN HORIZONTAL AND HOUR COORDINATE SYSTEMS

Vipin Kumar, Associate Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- vipinkumar2629@gmail.com

ABSTRACT:

The relationship between the Horizontal and Hour Angle Coordinate Systems is a fundamental concept in astronomy and celestial navigation. In this chapter, we explore how these two coordinate systems are interconnected and how they provide a powerful framework for tracking the motion of celestial objects across the night sky. The Hour Angle Coordinate System, based on the Earth's rotation, offers a means to express the position of objects with respect to the observer's meridian. Meanwhile, the Horizontal Coordinate System relates the altitude and azimuth of celestial bodies to the observer's local horizon. By understanding the intricate connection between these systems, astronomers and navigators can precisely locate and predict the movements of stars, planets, and other celestial phenomena.

This chapter elucidates the symbiotic relationship between the Horizontal and Hour Angle Coordinate Systems, making the complexities of celestial observation and navigation more accessible to both amateurs and experts we have delved into the fascinating relationship between the Horizontal and Hour Angle Coordinate Systems, shedding light on how they work in concert to aid astronomers, navigators, and stargazers in their quest to understand and navigate the celestial sphere.

KEYWORDS:

Spherical, Transformation, Triangular, Vertical, Zenith.

INTRODUCTION

The use of a coordinate system relies on how specialized the issue at hand is. Thus, the Equatorial Coordinate System is used to define positions of the stars regardless of the place of observation; it is the one used in catalogs and astronomical ephemeris; the Horizontal Coordinate System is used to obtain measures of the coordinates of stars; the Hour Coordinate System is used to point observational instruments in the desired direction; the Ecliptic Coordinate System is used to study the movements of objects in the Solar System; and the Galactic Coordinate System.

Although there are many additional coordinate systems, the five named are the most often used [1], [2].Finding formulae that enable transformations between different coordinate systems is important in order to tackle the numerous kinds of astronomical challenges. Both matrix rotation and the spherical trigonometry Gauss Group formulae may be used to perform these adjustments. Let's examine both of these approaches, beginning with the spherical trigonometry formulae, which some people may find simpler to understand. We link the Horizontal and Hour Coordinate Systems using the approach suggested in the preceding section. With the problem's data, we will first build a spherical triangle. Next, we will identify the known elements and the things we wish to compute [3], [4].The following components make up the spherical triangle.

$$a = 90 - h = z$$
$$b = 90 - \phi$$
$$c = 90 - \delta$$

where z is zenital distance and ø is the latitude of the observer; 6 is the object's decli- nation. The angles are

A0 = H

C = 180 – A

where H is the hour angle, and A the azimuth. Two situations can occur.

Hour from Horizontal

Consider a situation where we know the horizontal coordinates (z, A, ø) and we want to determine the hour coordinates (6, H). Applying the Fundamental Formula (Group I) to the spherical triangle we have:

cos (90◦ – 6) = cos (90 – h) cos (90◦ – ø) + sin (90 – h) sin (90◦ – ø) cos(180◦ – A) which is equal to sin 6 = sin h sin ø – cos h cos ø cos A (1.42)

Eq. (1.42) allows to determine the declination of the object. Now if we apply the law of sines (group V):

$$\frac{\sin(180^\circ - A)}{\sin(90^\circ - \delta)} = \frac{\sin H}{\sin z}$$

$$\sin H = \frac{\cos h \sin A}{\cos \delta}$$

Since the value of the hour angle (H) can be between 0◦ and 360◦, the value of the sine does not define it unequivocally and we need another function of H to obtain the quadrant where the object lies. This constrain comes from applying the five element formula (group III) to the triangle

sin (90 – 6) cos H = cos(90 – h) sin(90 – ø) – sin(90 – h) cos(90 – ø) cos(180 – A) cos H cos 6 = sin h cos ø + cos h sin ø cos A (1.44)

Equations Eq. (1.42), Eq. (1.43), and Eq. (1.44) solve the problem, allowing to obtain the Hour Coordinates (H, 6) from the local Horizontal coordinates (A, h). Summarizing:

$$\sin \delta = \sin h \sin \phi - \cos h \cos \phi \cos A$$
$$\sin H \cos \delta = \cos h \sin A$$
$$\cos H \cos \delta = \sin h \cos \phi + \cos h \sin \phi \cos A$$

First, we use the first equation to get sin 6 = 0.5132 (4 decimal places are sufficient to provide precision of 1/360000/=100). The reader should be made aware of a very crucial lesson as a result of the fact that there are only six sins, not seven. Declination is limited to the 4th and 1st quadrants since it ranges from 90° to 690°. The sign of the sine in this instance is clear, unlike the cosine, whose sign is never ambiguous since cos 6 is always positive [5], [6]. We can then find

$\delta = 30°52'52''$

We must use both the sine and cosine to clear up the quadrant ambiguity since the hour angle H ranges from 0 to 24 hours.

Sin A is negative since A = 240 is in the third quadrant. Since sin H in the second equation is negative, we may infer that H is either in the third or fourth quadrant. The cosine from the third equation, cos H = 0.4172, a positive number, will clear up the uncertainty.

H is thus either in the first or fourth quadrant.

We get to the conclusion that the fourth quadrant meets both sine and cosine signs, resulting in the hours H = 2990501100.

H = 19h 56m 21s.

So the coordinates of the star in the horizontal system are (H, 6)=(19h 56m 21s, 30◦5205200).

Horizontal from Hour

The inverse case is we know the hour coordinates and we want the horizontal coordi- nates. Applying the same group of formulae for the spherical triangle.

$$\sin h_{\max} = \sin\phi\sin\delta + \cos\phi\cos\delta \tag{1.49}$$
$$\cos h_{\max}\sin A = 0 \tag{1.50}$$
$$\cos h_{\max}\cos A = \sin\phi\cos\delta - \cos\phi\sin\delta \tag{1.51}$$

Eq. (1.49) leads to

$$\sin h_{\max} = \cos[\pm(\phi - \delta)] \tag{1.52}$$

So there are two possible maximum altitudes, $h_{\max} = 90 \pm (\delta - \phi)$. The ambiguity is to be resolved by the azimuth.

If an object with a 6 > 90 culminates to the south (-) or north (+) of the zenith, the altitude is positive. The star will never rise if the maximum altitude is negative.

At this latitude, objects with declinations below this critical declination are never visible. Using Apache Point Observatory (APO, 33) as an example, austral stars always reach their maximum south of the zenith.

Thus, 90° = 57° is the crucial declination, and stars more arid than this can never be seen [7], [8].

Circumpolar stars and the lowest altitude

When H = 12h, the height of a star is at its lowest. The formulae for the lowest altitude are as follows: An object's height is at its highest when it crosses the meridian. Its hour angle at that time (upper culmination) is H = 0h. Eq. (1.46), for H = 0h, yields its height:

$$\sin h_{\min} = \sin\phi\sin\delta - \cos\phi\cos\delta \quad (1.58)$$
$$\cos h_{\min}\sin A = 0 \quad (1.59)$$
$$\cos h_{\min}\cos A = -\sin\phi\cos\delta - \cos\phi\sin\delta \quad (1.60)$$

Eq. (1.58) leads to

$$\sin h_{\min} = -\cos[\pm(\phi+\delta)] = \sin[-90^\circ \pm (\phi+\delta)] \quad (1.61)$$

So there are two possible minimum altitudes, $h_{\min} = -90 \pm (\phi+\delta)$. The ambiguity is to be resolved by the azimuth.

Eq. (1.59) leads to sin A = 0, i.e, the star is either at A = 0◦ or A = 180◦, i.e, the meridian. Eq. (1.60) leads to,

$$\cos h_{\min}\cos A = -\sin(\phi+\delta) \quad (1.62)$$
$$= -\cos[90 - (\phi+\delta)] \quad (1.63)$$

The ambiguity is resolved: if $A = 180^\circ$ (the star has lower culmination north of the meridian), then

$$\cos h_{\min} = \sin(\phi+\delta) = \cos[90 - (\delta+\phi)] \quad (1.64)$$

that means, either $\pm h_{\min} = \pm 90 \mp (\delta+\phi)$ or $\pm h_{\min} = \mp 90 \pm (\delta+\phi)$. From the 1st condition we need to pick $h_{\min}$ and -90°, so,

$$h_{\min} = \delta + \phi - 90^\circ \quad (1.65)$$

for culmination north of the zenith. If, on the other hand, $A = 0^\circ$ and the star culminates south of the zenith, then

$$\cos h_{\min} = -\sin(\delta+\phi) = \cos[-90 - (\delta+\phi)] \quad (1.66)$$

and thus

$$h_{\min} = -90 - (\delta+\phi) \quad (1.67)$$

Summarizing, the minimum altitude of a star is

Summarizing, the minimum altitude of a star is:

$$h_{\min} = \begin{cases} -90 + (\delta+\phi) & \text{if A = 180 (lower culmination north of zenith)} \\ -90 - (\delta+\phi) & \text{if A = 0 (lower culmination south of zenith)} \end{cases}$$

If the object culminates to the north (+) or south (-) of the zenith, the minimum altitude is negative for objects with 90. The star won't set if the minimum altitude is positive. The term "circumpolar" refers to objects that are always above the horizon at latitude and have declinations greater than this crucial declination. Boreal stars, for instance, usually have lower culmination north of the zenith as seen from APO (33).

Therefore, the critical declination is 6 = 90 = 57. Any more polar stars than this are circumpolar [9], [10].

Rising and Setting Times

Isolating cos H in Eq. (1.46), we can find the hour angle of an object at the moment that its altitude is h

$$\cos H = -\tan\delta\tan\phi + \frac{\sin h}{\cos\delta\cos\phi} \qquad (1.69)$$

To determine rising and setting times, utilize this equation. The hour angles corresponding to rising and setting times are because rising and setting correspond to altitude h = 0:

cos H (h = 0) = – tan 6 tan ø   (1.70)

Relationship between Hour Coordinates and Celestial Equatorial coordinates

The equator serves as the fundamental plane for both the hour coordinate system and the equatorial coordinate system.

The relationship between hour angle (H) and right ascension (), or the abscissa, is all that is required since the ordinate is the same for both systems (declination 6).

The vernal point, which travels with the celestial sphere and finally coincides with the origin of the hour angle when it is in the meridian, is the source of right ascension. Except that the angle between the sources now is constantly changing. Let's use the letter T to represent the vernal point's hour angle,

$$\boxed{T = \alpha + H}$$

Equation Eq. (1.71) is regarded as the basic astrological formula. It has a fixed right ascension.

The value of T, which ranges from 0 to 24 hours, rises together with the hour angle H. T is the local sidereal time, which corresponds to the vernal point's hour angle in Figure 1's vernal point. Eq. (1.71) may be used to calculate the sidereal time T if the hour angle is observed and the right ascension is known.

The Local Sidereal Time may be used to quantify time. A sidereal day is the period of time between the vernal point's (ç) two successive transits across the meridian.

Using Eq. (1.71) we have

↲ = 7h 45m 00s – 5h 43m 24s = 2h 01m 36s (1.72)

We must measure time intervals using a sidereal clock if we wish to be precise. A sidereal clock runs 3 min. 56.56 seconds faster each day than a regular solar clock, thus 24 hours of solar time correspond to 24 hours and 3 min. 56 seconds of sidereal time. 23h 56m 04s of solar time is equivalent to 24 sidereal hours.

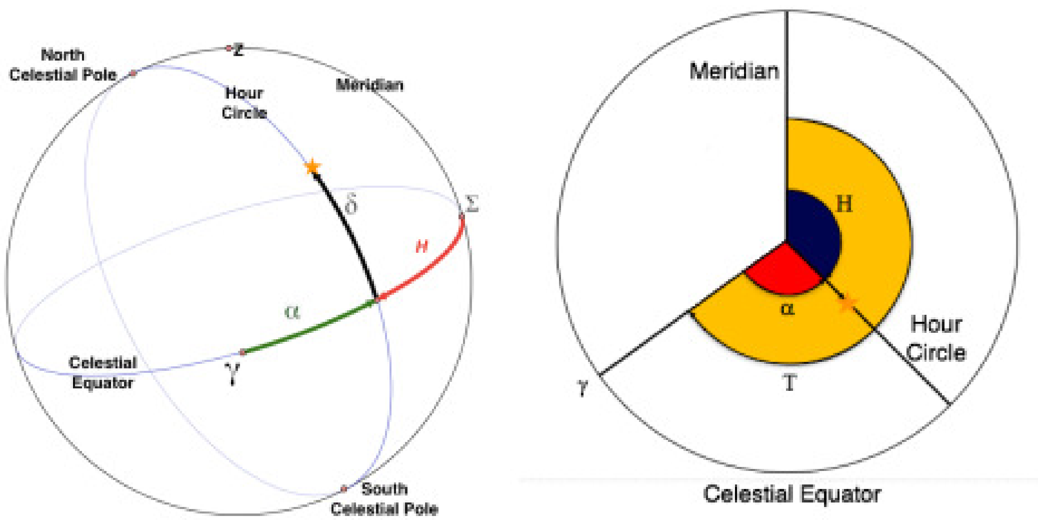


Figure 1: Conversion between equatorial and hour coordinate systems.

Because of how the Earth orbits the Sun, stars seem to move across the sky more quickly than the Sun, hence a sidereal clock must tick more quickly.

Relationship between Horizontal and Celestial Equatorial Co- ordinates

1st case: known z, A, ø, T =) unknown ↲, 6.

In order to obtain the equatorial coordinates it is enough to substitute $H = T - \alpha$ into equations Eq. (1.43) and Eq. (1.44)

$$\sin\delta = \sin h \sin\phi - \cos h \cos\phi \cos A \qquad (1.73)$$

$$\sin(T-\alpha)\cos\delta = \cos h \sin A \qquad (1.74)$$

$$\cos(T-\alpha)\cos\delta = \sin h \cos\phi + \cos h \sin\phi \cos A \qquad (1.75)$$

Relationship between Celestial Equatorial and Ecliptic

The ecliptic longitude h, calculated from the vernal point, and the ecliptic latitude are the coordinates in the ecliptic system. Let's build the triangle.

The spherical triangle is formed by the north celestial pole, the north ecliptic pole, and the star. Its sides are 90 degrees, 90 degrees, and 90 degrees, respectively, and its angles are 90 degrees and 90 degrees, respectively.

The first instance involves the cases of knowing 6 and s, as well as unknown h and.

The spherical triangle may be solved using the formulae from groups I, III, and V to get at:

$$\begin{aligned}
\sin\beta &= \cos\varepsilon\sin\delta - \sin\varepsilon\cos\delta\sin\alpha && (1.79)\\
\cos\beta\cos\lambda &= \cos\delta\cos\alpha && (1.80)\\
\cos\beta\sin\lambda &= \sin\varepsilon\sin\delta + \cos\varepsilon\cos\delta\sin\alpha && (1.81)
\end{aligned}$$

The second case is with known β, λ, and ε, while δ and α are unknown.

$$\begin{aligned}
\sin\delta &= \cos\varepsilon\sin\beta + \sin\varepsilon\cos\beta\sin\lambda && (1.82)\\
\cos\delta\cos\alpha &= \cos\beta\cos\lambda && (1.83)\\
\cos\delta\sin\alpha &= -\sin\varepsilon\sin\beta + \cos\varepsilon\cos\beta\sin\lambda && (1.84)
\end{aligned}$$

Since we already know the equatorial coordinates and need the ecliptic coordinates, let's utilize equations 1.79 and 1.81. By using Eq. (1.79), we first find the ecliptic latitude. Applying the numerical numbers, we obtain since there is no sign ambiguity in the sine of a latitude.

$$\beta = 46^\circ 04' 45''$$

Let's use equations 1.80 and 1.81 to calculate the value of the ecliptic longitude h. The value of h may be in the second or third quadrant since the cosine value in Eq. (1.80) is negative. In Equation (1.81), the sine value is positive, and h may be in either the first or second quadrant. As a result, the longitude value can only be in the second quadrant. So,

$$\lambda = 102^\circ 38' 19''.$$

The coordinates of the Sun

The Sun's equatorial coordinates are easily derived from the ecliptic equations. By definition, the Sun's ecliptic latitude is zero, or = 0. So,

$$\begin{aligned}
\sin\delta_\odot &= \sin\varepsilon\sin\lambda_\odot\\
\cos\alpha_\odot\cos\delta_\odot &= \cos\lambda_\odot\\
\sin\alpha_\odot\cos\delta_\odot &= \cos\varepsilon\sin\lambda_\odot
\end{aligned}$$

To leading order, the Sun's motion in ecliptic longitude can be parametrized as uniform, going 360◦/365 ⇡ 5901100 per day. So,

$$\lambda(t) \approx 0.986^\circ/\text{day} \times t(\text{days})$$

with t expressed as the number of days from the vernal equinox on March 22. When Eq. (1.90) is plugged into Eqs. (1.87)-Eqs. (1.89), the time functions S(t) and 6S(t) are produced.

Let's divide the issue into more manageable components. Let's start by calculating the Sun's ecliptic longitude. Since April has 30 days and March has 31 days, the number of days since the vernal equinox is t = May 27 - March 22 = 66. This means that there have been 9 days since the equinox, plus 30 days of April, plus 27 days of May, for a total of 66 days. Consequently, the Sun's ecliptic longitude is,

$$\lambda_{\odot} = \lambda'_{\odot} \times t = 0.986^{\circ}/\text{day} \times 66\,\text{days} = 65^{\circ}06'$$

Next, let's determine whether the Sun is above or below the ecliptic, or the sign of the declination. Since s is fixed at 232702600 and is in the first quadrant, sin s is positive, according to Eq. (1.87), the sign of sin 6S is the sign of sin hS. Sin hS is positive because the angle hS = 65060 is in the first quadrant. Sin 6S is likewise positive, hence it must be in the first or second quadrant according to Eq. (1.87). Declination can only be in the first or fourth quadrants since it ranges from -90 to 90 degrees. 6S is therefore located in the first quadrant. The Sun's declination is 6S = 2190 using Eq. (1.87).

Let's first define the quadrant before addressing the right ascension. hS, 6S, and s are all in the first quadrant according to equations (1.88) and (1.89), and because sin S and cos S are also positive, S is likewise in the first quadrant. The amount is,

$$\alpha = 26.84^{\circ} = 1\text{h}47\text{m}$$

It is important to note that the Sun's right ascension does not grow at a constant pace because cos S cos 6S = cos hS and the Sun travels in the ecliptic rather than at the equator.

The Sun's right ascension, declination, ecliptic latitude, and longitude are all zero during the vernal equinox, with values of S = 0, 6S = 0, lS = 0, and bS = 0. The derivative of the first equation yields the rate at which the Sun's declination grows:

$$\delta' = \left(\frac{\sin\varepsilon\cos\lambda}{\cos\delta}\right)\lambda'$$

When cos hS = 0, that is, when h = 90 or h = 270, it is zero. The Sun pauses and reverses direction at these locations. These are the solstices (Latin: sol, meaning sun, sistere, meaning to stop). The Sun's declination during the solstices is:

$$\delta = \pm\varepsilon$$

The term "tropics" (from the Greek, trope, to turn) refers to these lines of declination. The parallels of declination where the solstices occur are the tropics. The second equation, cos S = 0, yields the right ascension of the Sun at the solstice, together with S = 6 h and S=18 h.

CONCLUSION

The location of a celestial object may be determined over time using the Hour Angle Coordinate System, which is based on Earth's rotation. The Hour Angle gives a precise way to trace an object's journey across the sky as Earth spins and is the key to connecting the celestial equator to the observer's meridian. The Horizontal Coordinate System, which is closely related to an observer's position on Earth, was also looked at. This methodology provides a simple method to calculate an object's height and azimuth by connecting celestial objects to the observer's local horizon.

The Hour Angle is a useful tool for celestial navigation and stargazing because it allows observers to identify the location of any celestial object in the night sky by combining azimuth and altitude readings. We stress the significance of understanding the link between these coordinate systems as we draw to a close our investigation. Navigators may use this information to navigate on land and at water, while astronomers can use it to organize their observations precisely and record the marvels of the cosmos. It provides a window into the universe for astronomers and stargazers, enabling them to admire the constantly shifting cosmic tapestry overhead. The union of time and space is essentially represented by the connection between the Horizontal and Hour Angle Coordinate Systems, which makes the celestial sphere accessible to anybody who looks up in awe and inquiry.

REFERENCES:

[1] X. Y. Zeng, L. Shu, G. M. Huang, and J. Jiang, "Triangular fuzzy series forecasting based on grey model and neural network," *Appl. Math. Model.*, 2016, doi: 10.1016/j.apm.2015.08.009.

[2] K. Fadhloun, H. Rakha, and A. Loulizi, "Analysis of moving bottlenecks considering a triangular fundamental diagram," *Int. J. Transp. Sci. Technol.*, 2016, doi: 10.1016/j.ijtst.2017.01.003.

[3] M. Semisch, E. Hagert, M. Garcia-Elias, A. Lluch, and S. Rein, "Histological assessment of the triangular fibrocartilage complex," *J. Hand Surg. Eur. Vol.*, 2016, doi: 10.1177/1753193415618391.

[4] S. P. Shen *et al.*, "Quantum electric-dipole liquid on a triangular lattice," *Nat. Commun.*, 2016, doi: 10.1038/ncomms10569.

[5] W. Zhang *et al.*, "Giant facet-dependent spin-orbit torque and spin Hall conductivity in the triangular antiferromagnet IrMn3," *Sci. Adv.*, 2016, doi: 10.1126/sciadv.1600759.

[6] Y. Shen *et al.*, "Evidence for a spinon Fermi surface in a triangular-lattice quantum-spin-liquid candidate," *Nature*, 2016, doi: 10.1038/nature20614.

[7] J. K. Andersson, P. Axelsson, J. Strömberg, J. Karlsson, and J. Fridén, "Patients with triangular fibrocartilage complex injuries and distal radioulnar joint instability have reduced rotational torque in the forearm," *J. Hand Surg. Eur. Vol.*, 2016, doi: 10.1177/1753193415622342.

[8] R. Sevilla, L. Rees, and O. Hassan, "The generation of triangular meshes for NURBS-enhanced FEM," *Int. J. Numer. Methods Eng.*, 2016, doi: 10.1002/nme.5247.

[9] J. Shao and L. You, "On some properties of three different types of triangular blocked tensors," *Linear Algebra Appl.*, 2016, doi: 10.1016/j.laa.2016.09.001.

[10] J. Wang, D. Ding, O. Liu, and M. Li, "A synthetic method for knowledge management performance evaluation based on triangular fuzzy number and group support systems," *Appl. Soft Comput. J.*, 2016, doi: 10.1016/j.asoc.2015.09.041.

CHAPTER 4

A REVIEW STUDY OF SUN AT THE ZENITH: TROPICS

Abhinav Saxena, Associate Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- abhinav.polytechnic@tmu.ac.in

ABSTRACT:

The phenomenon of the Sun reaching the zenith in the tropics is a captivating celestial event that plays a pivotal role in the lives of those living within the tropical regions of our planet. In this chapter, we embark on a journey to explore the scientific, cultural, and practical significance of this natural occurrence. We delve into the mechanics of how the Sun attains its zenith position, leading to the awe-inspiring moment when it appears directly overhead. We examine the cultural and historical implications of this event, tracing its influence on indigenous cultures, traditions, and the way of life in the tropics. Additionally, we shed light on the practical aspects, including the implications for solar energy generation, agriculture, and the rhythms of daily life. By the chapter's end, readers will have gained a profound appreciation for the Sun at the zenith and its multifaceted impact on the tropical world. we have embarked on a journey to understand the Sun at the zenith in the tropics, an event that bridges the realms of science, culture, and practicality. We began by unraveling the scientific mechanics behind the Sun reaching its zenith position. We explored how this phenomenon occurs when the Sun is at its highest point in the sky, directly overhead, in regions situated between the Tropic of Cancer and the Tropic of Capricorn. The intricate interplay of Earth's axial tilt and its orbit around the Sun creates this remarkable celestial event, marking the zenith as a unique feature of tropical geography.

KEYWORDS:

Equatorial, Geographical, Geographic, Horizontal, Longitude, Polar.

INTRODUCTION

Consider the following problems:

1. At what latitude will the Sun culminate with h = 90◦ at the solstice [1], [2].

Using the equation for altitude with 6S = s and at culmination (H = 0, A = 0).

$$\sin 90^\circ = \sin\phi\sin\varepsilon + \cos\phi\cos\varepsilon = \cos(\phi-\varepsilon) \qquad (1.95)$$
$$\cos 90^\circ = \sin\phi\cos\varepsilon - \cos\phi\sin\varepsilon = \sin(\phi-\varepsilon) \qquad (1.96)$$

i.e., $\phi = \varepsilon$.

2. From what latitudes can the Sun be seen at the zenith in culmination?

$$\sin h = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H \qquad (1.97)$$
$$\cos h\sin A = \cos\delta\sin H \qquad (1.98)$$
$$\cos h\cos A = \sin\phi\cos\delta_\odot\cos H - \cos\phi\sin\delta \qquad (1.99)$$

$$\sin 90^\circ = \cos(\phi-\delta_\odot) = 1 \qquad (1.100)$$
$$\cos 90^\circ = \sin(\phi-\delta_\odot) = 0 \qquad (1.101)$$

that is, from any place where = 6S exists. There are locations on Earth where the Sun may be seen at its zenith because the Sun's declination ranges from -s to s. The intertropical zone is defined by these 3. The Sun's circumpolar position on Earth is determined by what latitudes? We must specify the conditions under which the Sun is always visible, that is, while its minimum height, hmin, is still positive. Given a location during the summer solstice in the northern hemisphere [3], [4].

$$\phi = 90^\circ - \varepsilon = 66^\circ 33'$$

In the northern hemisphere, this describes the arctic circle, whereas in the southern hemisphere, it defines the Antarctic circle. Day length as a function of the Sun's declination Find the amount of time the Sun is in the sky to determine the length of the day.

$$t = 2\cos^{-1}(-\tan\delta_\odot \tan\phi)$$

At the equinox (6S = 0) the duration of the day is

$$t = 2\cos^{-1} 0 = \pi \text{ radians} = 12\text{h}$$

So, days and nights have equal duration. This is the origin of the name equinox (equal night). We take the derivative of Eq. (1.103) with respect to 6 to find when the day is maximum or minimum.

$$t' = -\frac{2}{\sqrt{1-(\tan^2\delta_\odot \tan^2\phi)}}\left(\frac{\tan\phi}{\cos^2\delta_\odot}\right)\delta'_\odot \qquad (1.105)$$

Twilight

The transitional period between day and night is known as twilight, during which it is still bright outdoors although the Sun has set. Twilight happens when sunlight illuminates the lower atmosphere and is reflected and scattered by Earth's higher atmosphere. sunrise is a common name for sunrise in the morning and dusk in the evening. Based on the Sun's elevation, astronomers categorize twilight into three distinct phases [5], [6]. These twilights are civil, nautical, and astronomical.

Civil twilight: –6◦ < hS<0◦.

Nautical twilight: –12◦ < hS <–6◦.

Astronomical twilight –18◦ < hS<–12◦.

Civil twilight

When the Sun is less than 6 degrees below the horizon, civil twilight is present. Civil twilight in the morning lasts till dawn and starts when the Sun is 6 degrees below the horizon. It starts at sunset and finishes when the Sun is 6 degrees below the horizon in the evening. Civil dawn occurs when the Sun's geometric center is 6 degrees below the morning horizon. Civil dusk occurs when the Sun's geometric center is 6 degrees below the horizon in the late afternoon or early evening. The brightest twilight is civil twilight. During this time, there is enough natural light to do outside activities without the need for artificial lighting. During this time, only the brightest celestial objects may be seen with the unaided eye. This concept of civil twilight is used by several nations to create regulations governing hunting, flying, and the use of city lights [7], [8].

Maritime twilight

When the Sun's geometric center is between 6 and 12 degrees below the horizon, nautical twilight is present. Artificial light is often needed for outdoor activities during this twilight time since it is less bright than civil twilight. The stars and horizon are visible during the nautical twilight, making it a suitable time for observations. When the Sun is 12 degrees below the horizon in the morning, nautical dawn occurs. When the Sun dips 12 degrees below the horizon in the evening, nautical dusk begins. When sailors used the stars to chart their course across the ocean, the phrase "nautical twilight" first appeared. The majority of stars are readily visible at this hour with the unaided eye.

Astronomical Dusk

When the Sun is between 12 and 18 degrees below the horizon, astronomical twilight occurs. When the Sun is 18 degrees below the horizon, it is considered to be astronomical dawn. The sky is completely black before this. Astronomical nightfall occurs at the precise moment when the Sun's geographic center is 18 degrees below the horizon. The sky is no longer lighted after this. The sky is entirely black in the morning before the start of astronomical twilight, and in the evening after the conclusion of astronomical twilight. After this phase is through, any celestial objects that are visible to the unaided eye may be seen in the sky.

Length of the dusk

Twilight's duration varies with latitude. Twilight periods are often shorter in equatorial and tropical areas than they are in places at higher latitudes. In the summer, there may not be a difference between astronomical twilight before dawn and twilight after sunset at higher latitudes. This occurs when the Sun's height at local midnight is between 18 and 0. Similar to this, higher latitudes may experience a protracted period of nautical twilight if the Sun doesn't dip below the horizon by more than 12 degrees. If the Sun is less than 6 degrees below the horizon for the whole night, even higher latitudes enjoy a prolonged period of civil twilight [9], [10].

There is no astronomical or nautical twilight at the North Pole for a few days before the March equinox. In its place, civil twilight prevails continuously. At the North Pole, the Sun rises at the equinox and remains there throughout the day until the September equinox. The North Pole doesn't experience any twilight at this time. The Midnight Sun or Polar Day are two names for this event. When the Sun dips below the horizon a few days after the September equinox, the North Pole experiences many days straight of solely civil twilight, followed by days of nautical twilight and eventually astronomical twilight. When the Sun descends more than 18 degrees below the horizon in October, this transition will have ended. When this occurs, the pole goes through a period of darkness without twilight known as Polar Night. Astronomical twilight is visible to observers on the North Pole by the beginning of March. As the Sun climbs higher in the sky, there are a few days of nautical twilight after that. At the South Pole, the same phenomenon may be seen, although at different periods of the year.

Longitude

Science has struggled with determining longitude for a very long time. Time may be used to compute longitude since a day has 24 hours. The longitude of 15 degrees equates to one hour of time difference (360/24 = 15/hour). Assume that the observer sets their watch to 12 o'clock at noon in Greenwich before departing a long way. The spectator then realizes that, according to their clock, the Sun is at its greatest point in the sky at 4 PM. When this happens, the observer knows they are in longitude 60 W (4 hours 15 minutes every hour = 60). Time is length. It should be noted that noon does not, in general, imply 12:00 PM as shown by your watch. Instead, it is when the Sun is at its highest point in the sky. The majority of us do not reside on the time zone meridian since time zones were officially determined at a period when every city observed its own local time. The Sun will be at its highest point in the sky for an observer due to the usage of time zones between around 11:30 AM and 12:30 PM local time, or later if daylight saving time is used. The time of genuine astronomical noon may be determined if longitude is known.APO, for instance, is located at longitude 105.8197 W, which is 118.2437/ (15 hours) = 7.0546 hours. Due to the fact that APO is on Mountain Time and is 7 hours from Greenwich, astronomical noon occurs 3 minutes after 12 PM (7.0546 - 7) hours. So, at APO, "noon" is around 12:03 PM. In general, the Sun and noon are not necessary. Any star, at any hour angle, will do.

$$L = \text{Local Time} - \text{Greenwich Time}$$

Astronomical Time

Sidereal and Solar time

The hour angle of the vernal equinox is known as sidereal time. This allows us to define the sidereal day, which is the period of time between the vernal point's two sequential culminations in Figure 1. The solar day, also known as the synodic day, is 3 minutes, 56.56 seconds longer than the sidereal day due to the motion of the Earth in its orbit. The solar and sidereal days are in phase and synchronize after the Earth has completed one complete orbit. Thus, a sidereal year has a day that the solar year does not. If we write the solar day as P/☒S and the sidereal day as P/☒x

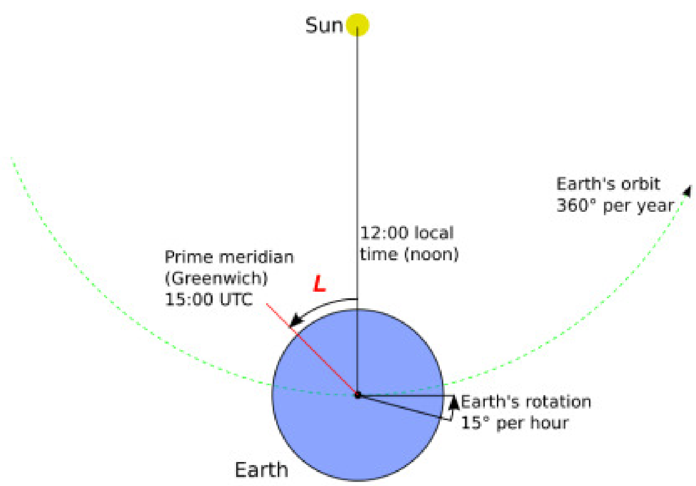


Figure 1: Longitude is the angle between a predefined meridian (Greenwich) and your local meridian.

$$\frac{P}{\tau_\star} = \frac{P}{\tau_\odot} + 1 \qquad (1.108)$$

$$\frac{1}{\tau_\odot} = \frac{1}{\tau_\star} - \frac{1}{P} \qquad (1.109)$$

Mean Solar Time and the Equation of Time

We sentient creatures on the surface of planet Earth have long relied on the Sun to keep track of time and control daytime activities because of the stark contrast between day and night. Find a symmetry is an excellent general rule. The Sun's highest height was measured at the moment it crossed the meridian since rising and setting periods change throughout the year. This would be noon, and the time before and after this point in the day could be evenly split (ante meridian, AM; post meridian, PM).However, if sufficient precision is needed in Figure 2, it might be challenging to define time based on the Sun. When the Sun has a zero-hour angle, it is at its meridian passage. It relies on the Sun's right ascension according to Eq. (1.71). The Sun moves at the ecliptic, not the equator, which is the first difficulty. This means that its shift in right ascension is not consistent with its change in ecliptic longitude. We need to solve Eq. (1.87)-Eq. (1.89), together with:

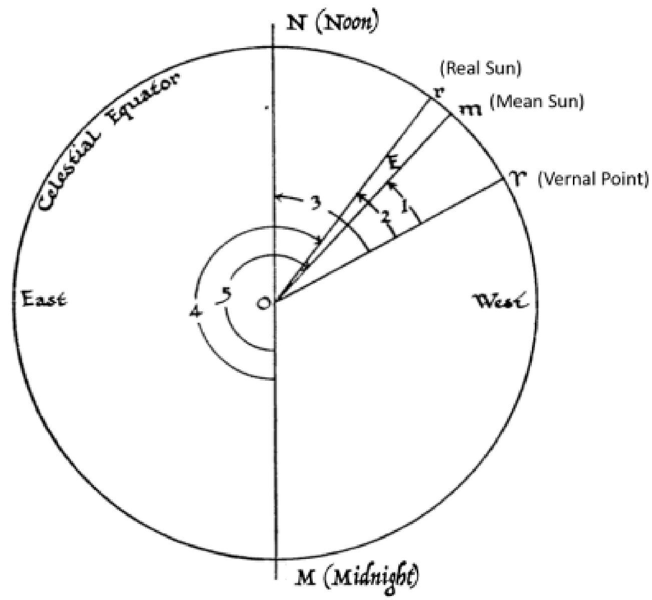


Figure 2: Polar projection. The angle 1 is the right ascension of the mean Sun. The angle 2 is the right ascension of the true Sun. The angle 3 is the local sidereal time. The angle 4 is the true solar time, and the angle 5 is the mean solar time.

The second issue is that Eq. (1.90) can only be a rough estimate. We considered the ecliptic motion of the Sun to be uniform. If the Earth's orbit were round, only then would this be accurate. Since the Sun's ecliptic motion is elliptical, it mirrors the Earth's orbital speed: from our vantage point, the Sun moves the quickest at perihelion (close to the December solstice) and the slowest at aphelion (close to the June solstice).

We may define a mean Sun (Figs. 1.22 and 1.23), which is a hypothetical Sun that travels along the celestial equator at a constant speed (0 const), to prevent these non-uniformities. This fictitious Sun passes past the vernal equinox twice in one tropical year (or solar year), or exactly 24 hours apart (the mean solar day), and exactly 24 hours of right ascension. Because of precession, this differs from the sidereal year, which is the period between two crossings of the Sun by the same star. The sidereal year is 365.2564 days; the tropical year is 365.2422 days.The hour angle of the mean Sun + 12 hours (so that the date changes at the lower culmination, midnight, rather than at noon) is how the mean Sun establishes a mean solar time (or mean time) TM.

$$T_M = H_M + 12\text{h}$$

The genuine solar time, which closely follows the daily motion of the actual Sun, is inconvenient to mean solar time. By determining the Sun's hour angle, you may determine the real solar time. True solar noon occurs when the Sun crosses the observer's meridian. The time between two consecutive meridian passes is the genuine solar day.

The equation of time describes the difference between the real solar time and the meantime TM.

$$E = T - T_{\text{M}} \quad (1.112)$$

Here "equation" is used in the sense of "reconciling a deference".

Every day at the same time, say 3 p.m. as shown by your wall clock set to civil time (solar mean time), note the Sun's location. The so-called analemma will have been noted by you. The figure looks like a thin 8. The yearly fluctuation in the Sun's declination is what causes the north-south variation. The ecliptic tilt and eccentricity of Earth's orbit both affect right ascension, which is what causes the east-west tilt. The equation of time accounts for this east-west motion.To get the mean solar time, the true solar time T (hour angle of the Sun + 12 h) must be adjusted by ET. If local time zones weren't based on political boundaries, this imply solar time would be the same as the time shown on your watch. The Greenwich Meridian is used as a worldwide reference and is referred to as Universal Time in order to create a standard reference.The Prime Meridian in Greenwich, London, United Kingdom, measures mean solar time, making it the source of the term "Universal Time" (GMT). The synchronized universal time, or UTC, used in contemporary wall clocks is determined by atomic processes.

Sidereal Time to Civil Hour

The time zone's correction for UTC is the civil hour. From -12 through 0 (GMT) to +12, there are 25 integer World Time Zones. As measured East and West from the Prime Meridian at Greenwich, each one is 15 of longitude. The modifications are as follows to convert apparent solar time: To get the mean solar time, correct the longitude with respect to the time meridian. We previously performed it for APO and discovered a + 3-minute adjustment. So, UTC - 7h + 3m is the mean solar time. Subtract the Equation of Time to get genuine solar time from mean time. Consider that the sidereal day is 30 55.909 seconds shorter than the solar day, or = 23 hours, 56 minutes, 4.091 seconds, in order to get the sidereal time.

Sidereal Day = Solar Day – 3m 55.909s (1.113)

They synchronize on the vernal equinox. The true astronomical noon at the Vernal Equinox is 00:00 hours local sidereal time. They will be slower by 0.0655 h a day. 185 days after the vernal equinox, the mismatch will be 0.0655 185 12 h. At this time, the autumnal equinox, sidereal time synchronizes with solar time (as civil day changes at midnight and astronomical day at noon).

Sidereal Day = Solar Day – 3m 55.909s

We must first adjust for longitude. While Cape Town observes GMT+2, its longitude, 18.49E, puts it 1 hour, 13 minutes, and 58 seconds ahead of Greenwich. O by 0.7673 h, or 46m 02s, is 3h civil. The mean solar time at 3 p.m. is TM = 14h13m58s. Then, we convert to genuine solar time T using the equation of time. According to the graph, late July's adjustment amounts to around 6.5 minutes, which must be added to TM in order to get T. The time is T 14h 20m 30s. After subtracting 12 hours (sidereal time changes at noon and civil time at midnight), we may convert to sidereal time)

$$H = T - 12h = 2h\ 20m\ 30s \quad (1.114)$$

and account for the shift from the solar to the sidereal day. The period from March 22 to July 24 was 124 days. Make it 123 because we erased 12 hours. The adjusted hours are 0.0655 x 123, which is 8.0565. The sidereal time is thus 10.3982 = 10h 23m 53s.

CONCLUSION

We discovered that the Sun's position at its peak has a significant impact on indigenous cultures and customs in tropical areas from both a cultural and historical standpoint. This event has had a profound impact on the lives of those who live in the tropics, from celebrations and ceremonies honoring the Sun's strength to the creation of architecture that takes into account the zenith's impacts. Finally, we looked at the actual effects of the Sun being at its highest point. It is a great resource for solar energy production because to its constancy and intensity, which is advantageous for tropical locations looking for environmentally friendly power sources. The style of life in these locations is also influenced by its effects on agriculture, everyday routines, and outdoor activities. We ask readers to reflect on how deeply interwoven nature, society, and science are as we wrap up our examination of the Sun at its maximum. This celestial occurrence not only serves as a reminder of the dynamic interaction between our planet and the Sun, but it also sheds light on the great diversity of human experience around the globe. Whether seen through the eyes of science or the prism of culture, the Sun at its maximum is a tribute to the glories of the natural world and the lasting effect it has on our lives.

REFERENCES:

[1] S. N. Volkov, I. V. Samokhvalov, H. Du Cheong, and D. Kim, "Optical model and calibration of a sun tracker," *J. Quant. Spectrosc. Radiat. Transf.*, 2016, doi: 10.1016/j.jqsrt.2016.04.020.

[2] J. Lin, Z. Lee, M. Ondrusek, and K. Du, "Remote sensing of normalized diffuse attenuation coefficient of downwelling irradiance," *J. Geophys. Res. Ocean.*, 2016, doi: 10.1002/2016JC011895.

[3] C. Robles-Gonzalez *et al.*, "Intercomparison of stratospheric nitrogen dioxide columns retrieved from ground-based DOAS and FTIR and satellite DOAS instruments over the subtropical Izana station," *Atmos. Meas. Tech.*, 2016, doi: 10.5194/amt-9-4471-2016.

[4] P. M. López-Serrano, J. J. Corral-Rivas, R. A. Díaz-Varela, J. G. álvarez-González, and C. A. López-Sánchez, "Evaluation of radiometric and atmospheric correction algorithms for aboveground forest biomass estimation using landsat 5 TM data," *Remote Sens.*, 2016, doi: 10.3390/rs8050369.

[5] Z. Ni *et al.*, "Investigation of atmospheric effects on retrieval of sun-induced fluorescence using hyperspectral imagery," *Sensors (Switzerland)*, 2016, doi: 10.3390/s16040480.

[6] Y. Chu, M. Li, and C. F. M. Coimbra, "Sun-tracking imaging system for intra-hour DNI forecasts," *Renew. Energy*, 2016, doi: 10.1016/j.renene.2016.05.041.

[7] M. Shimoni, R. Haelterman, and P. Lodewyckx, “Advancing the retrievals of surface emissivity by modelling the spatial distribution of temperature in the thermal hyperspectral scene,” in *Algorithms and Technologies for Multispectral, Hyperspectral, and Ultraspectral Imagery XXII*, 2016. doi: 10.1117/12.2223749.

[8] I. N. Williams, W. J. Riley, L. M. Kueppers, S. C. Biraud, and M. S. Torn, “Separating the effects of phenology and diffuse radiation on gross primary productivity in winter wheat,” *J. Geophys. Res. Biogeosciences*, 2016, doi: 10.1002/2015JG003317.

[9] J. Gao and J. Tang, “Research on the performance-based parameter of horizontal visor of certain wall relative to the sun zenith angle,” *Int. J. Multimed. Ubiquitous Eng.*, 2016, doi: 10.14257/ijmue.2016.11.4.27.

[10] A. P. R Srinivas and A. Professor, “Design and Development of a SOLAR TREE,” *Int. J. Sci. Eng. Res.*, 2016.

CHAPTER 5

EXPLORING THE INTRIGUING WORLD OF COORDINATE CHIRALITY

Alok Kumar Gahlot, Assistant Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- dr.alokgahlot02@gmail.com

ABSTRACT:

Coordinate chirality is a concept that transcends traditional Cartesian systems, introducing a unique perspective on spatial orientation and symmetry. In this chapter, we explore the intriguing world of coordinate chirality, where spatial coordinates are endowed with a sense of handedness. We delve into the fundamentals of chirality and its implications for various scientific fields, from chemistry and physics to biology and materials science. Throughout this journey, we uncover the significance of chirality in understanding complex molecules, particles, and structures, shedding light on its role in everything from drug development to quantum physics. By the chapter's end, readers will have gained a deeper appreciation for the nuanced dimension that coordinate chirality adds to our understanding of the world we embarked on a fascinating exploration of coordinate chirality, a concept that adds a profound layer of understanding to the symmetries and properties of spatial coordinates. We began by defining chirality as the property of having a non-superimposable mirror image, a concept deeply rooted in geometry and symmetry. Unlike traditional coordinates that exist in a mirror-symmetric world, chiral coordinates introduce a sense of handedness, where the left and right sides cannot be perfectly overlaid. This seemingly abstract notion carries significant implications in various scientific disciplines.

KEYWORDS:

Chiral, Coordinates, Geometry, Mirror-Image, Orientation.

INTRODUCTION

If a coordinate system's axes are orientated like the blue axes the right hand's index, middle, and thumb are all lined up with the x, y, and z axes, respectively it is direct, counterclockwise, or right-handed. If the axes of the coordinate system are facing the red axes, the coordinate system is indirect, clockwise, or left-handed. The left hand's thumb, middle finger, and index finger are each lined up with one of the x0, y0, or z0 axes. The following five coordinate systems are categorized as Left-handed: Horizontal hour [1], [2].

Ecliptic, equatorial, and galactic are all right-handed

The matrices described below are used to convert between right- and left-handed systems. Consider two coordinate systems that are left-handed (x0, y0, z0) and right-handed (x0, y0, z0), respectively. In matrix notation, it will be possible to achieve passing from the system (x, y, and z) to the system (x0, y, and z0).

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Similarly, passing from the system $x \equiv x'$; $y \equiv -y'$; $z \equiv z'$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1.124}$$

Finally, passing from the system $x \equiv -x'$; $y \equiv y'$; $z \equiv z'$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1.125}$$

We recall also the rotation matrices

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \tag{1.126}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \tag{1.127}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.128}$$

When a right-handed system is rotated counterclockwise from the positive end of the axis that it will be turned around, the angle is deemed positive. The angle will be seen negatively in clockwise rotations. The rotation of an angle for a left-handed system will be positive if done clockwise and negative if done counterclockwise. The rotating system itself must be used to measure the revolutions. The alternatives for the rotation angle's sign are shown in the table below [3], [4].

Matrix rotations to transform across coordinate systems

Instead of employing spherical trigonometry formulas, we may translate between coordinate systems using linear algebra. The primary benefit of linear algebra is that it makes it easier to employ computers to do the required computations.

Relationship between the Hour system and the Local Horizontal System

Let (x, Y, Z) be the Horizontal System coordinates and (x0, Y, Z) be Hour System coordinates. Then, we must rotate around the y axis 90 degrees in a counterclockwise direction to go from the horizontal to the hour coordinate system. Since the rotation is counterclockwise and the system is left-handed, the angle will be negative, or -(90). We have a matrix notation, which:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{H,\delta} = R_y[-(90^\circ - \phi)] \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{A,z} \tag{1.129}$$

$$\begin{pmatrix} \cos\delta\cos H \\ \cos\delta\sin H \\ \sin\delta \end{pmatrix} = \begin{pmatrix} \sin\phi & 0 & \cos\phi \\ 0 & 1 & 0 \\ -\cos\phi & 0 & \sin\phi \end{pmatrix} \begin{pmatrix} \sin z\cos A \\ \sin z\sin A \\ \cos z \end{pmatrix} \tag{1.130}$$

$$\begin{pmatrix} \cos\delta\cos H \\ \cos\delta\sin H \\ \sin\delta \end{pmatrix} = \begin{pmatrix} \sin\phi\sin z\cos A + \cos\phi\cos z \\ \sin z\sin A \\ -\cos\phi\sin z\cos A + \sin\phi\cos z \end{pmatrix} \tag{1.131}$$

For the conversion of the hour coordinate system into horizontal we perform a rotation in the clockwise direction. As the system is left-handed, the angle will be positive +(90◦ – ø). Then,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{A,z} = R_y(90^\circ - \phi)\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{H,\delta} \qquad (1.132)$$

and

$$\begin{pmatrix} \sin z \cos A \\ \sin z \sin A \\ \cos z \end{pmatrix} = \begin{pmatrix} \sin\phi\cos\delta\cos H - \cos\phi\sin\delta \\ \cos\delta\sin H \\ \cos\phi\cos\delta\cos H + \sin\phi\sin\delta \end{pmatrix} \qquad (1.133)$$

Relationship between Hour coordinates and Equatorial co- ordinates

The celestial equatorial coordinates are (x0, y0, z0) and the hour coordinates are (x, y, z). Since declination is used by both systems, a simple rotation around the z-axis will do. Keep in mind that the hour angle H's direction and the right ascensions are mutually exclusive. As a result, to transform from hour coordinates to equatorial, we must rotate by clockwise turning an angle T around the z-axis (x00, y00, z00). The angle (+T) is positive since the system is left-handed. Then, we must go from the left-handed (hour) system to the right-handed (equatorial) one).

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{\alpha,\delta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} R_z(T)\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{H,\delta} \qquad (1.134)$$

Similar to this, in order to go from equatorial to hour coordinates, we must first change from a right-handed to a left-handed system. Next, we must rotate by an angle T in the opposite direction, making the angle negative,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{H,\delta} = R_z(-T)\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{\alpha,\delta} \qquad (1.135)$$

Relation between Equatorial and Ecliptic coordinates

We must rotate the z-axis counterclockwise around the x-axis in order to get the ecliptic coordinates (x0, y0, z0) from the celestial equatorial coordinates (x, y, z). Due to the right-handed nature of the equatorial system, the angle will be positive (+s). Then,

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{\lambda,\beta} = R_x(\varepsilon)\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta} \qquad (1.136)$$

To pass from ecliptic to equatorial coordinates we simply rotate an angle (s) clockwise around the x0 axis. The angle will be negative because the ecliptic coordinate system is right-handed [5], [6].

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta} = R_x(-\varepsilon)\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{\lambda,\beta} \qquad (1.137)$$

Relation between the Celestial and Galactic Equatorial Systems

In order to convert the celestial equatorial coordinates (x, y, and z) into the galactic coordinates (x0, y0, and z0), we must: Start by rotating the z-axis counterclockwise by an angle of (). The rotation is in favor since the equatorial system is right-handed. Then, by an angle of i, we turn counterclockwise around the x-axis. The rotation is in the positive because the equatorial system is right-handed. An angle (l) is used to perform a third rotation of the z-axis in a clockwise direction. Right-handed systems' rotation will be counterclockwise [7], [8]. Then

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{l,b} = R_z(-l_\Omega)\,R_x(+i)\,R_z(+\alpha_\Omega)\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta} \qquad (1.138)$$

To obtain the celestial equatorial coordinates from the galactic coordinates simply reverse the path,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta} = R_z(-\alpha_\Omega)\,R_x(-i)\,R_z(+l_\Omega)\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{l,b} \qquad (1.139)$$

Coordinate Transformation by Translation

Now consider that the coordinate systems really differ in their origins as opposed to just their orientations. The matrix determines where a point P on a sphere's surface is located in a system S with an origin O.

$$\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = r\begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ \sin\theta \end{pmatrix} \qquad (1.140)$$

In respect to an S 0 system, with origin in O0, the position of the same point is given by

$$\boldsymbol{r}' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = r'\begin{pmatrix} \cos\theta'\cos\phi' \\ \cos\theta'\sin\phi' \\ \sin\theta' \end{pmatrix} \qquad (1.141)$$

The position of origin O0 with respect to O is given by the matrix:

$$\boldsymbol{R} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R\begin{pmatrix} \cos\Theta\cos\Phi \\ \cos\Theta\sin\Phi \\ \sin\Theta \end{pmatrix} \qquad (1.142)$$

If the axes of S 0 are parallel to their counterparts of S we then have

r = R + r0 (1.143)

or

$$r\begin{pmatrix}\cos\theta\cos\phi\\ \cos\theta\sin\phi\\ \sin\theta\end{pmatrix} = R\begin{pmatrix}\cos\Theta\cos\Phi\\ \cos\Theta\sin\Phi\\ \sin\Theta\end{pmatrix} + r'\begin{pmatrix}\cos\theta'\cos\phi'\\ \cos\theta'\sin\phi'\\ \sin\theta'\end{pmatrix} \qquad (1.144)$$

In case the change of reference frame requires translation and rotation, we first do translation and then rotation.

DISCUSSION

The transition of coordinate systems from those based on the location of observation (topocentric systems), to those based on the center of the Earth (geocentric systems), and eventually to those based on the solar system's barycenter (barycentric systems) was driven by the hunt for inertial systems.

The celestial equatorial and ecliptic coordinate systems, which have their origins in the center of the Earth, are really moved to the solar system's barycenter by executing a translation of the origin [9], [10].

Give the coordinate system (x0, y0, z0) an origin at the Earth's center, E. Let (x, y, z) be a coordinate system with an origin located at the solar system's barycenter, or S. Let (X, Y, Z) now represent the coordinates of the Solar System's barycenter as seen from the Earth's center.

The generic point P's geocentric coordinates may be expressed as:

$$\begin{aligned} x' &= X + x \\ y' &= Y + y \\ z' &= Z + z \end{aligned}$$

Additionally, the solar system's barycenter's geocentric coordinates, which are calculated from the celestial equatorial coordinates, are.

X = R cos 6⇥ cos ↵⇥ (1.148)

Y = R cos 6⇥ sin ↵⇥ (1.149)

Z = R sin 6⇥ (1.150)

where (↵⇥, 6⇥) are the barycenter's geocentric equatorial coordinates, and R is the astronomical unit-based separation between the barycenter and the planet's center. Commonly seen in ephemerides, the coordinates X, Y, and Z are calculated precisely based on the positions of all the planets. The geocentric equatorial coordinates may also be used to indicate the geocentric coordinates of point P:

$$\begin{aligned} x' &= \rho\cos\delta'\cos\alpha' \\ y' &= \rho\cos\delta'\sin\alpha' \\ z' &= \rho\sin\delta' \end{aligned}$$

where (↵0, 60) are the geocentric equatorial coordinates of point P and ⇢ is the distance from point P to the center of the Earth in astronomical units. The inverse relations are,

$$\rho = \left(x'^2 + y'^2 + z'^2\right)^{1/2} \quad (1.154)$$

$$\tan\alpha' = y'/x' \quad (1.155)$$

$$\tan\delta' = z'/\left(x'^2 + y'^2\right)^{1/2} \quad (1.156)$$

The barycentric coordinates of point P are

$$x = r\cos\delta\cos\alpha \quad (1.157)$$

$$y = r\cos\delta\sin\alpha \quad (1.158)$$

$$z = r\sin\delta \quad (1.159)$$

where (, 6) are the point P's barycentric equatorial coordinates and r is the distance, in astronomical units, between point P and the solar system's barycenter. In order to get the barycentric coordinates of point P, one must substitute Eqs. (6.181), (1.133), (6.186), and (1.131):

$$\cos\delta\cos\alpha = \frac{\rho}{r}\cos\delta'\cos\alpha' - \frac{R}{r}\cos\delta_\Theta\cos\alpha_\Theta \quad (1.160)$$

$$\cos\delta\sin\alpha = \frac{\rho}{r}\cos\delta'\sin\alpha' - \frac{R}{r}\cos\delta_\Theta\sin\alpha_\Theta \quad (1.161)$$

$$\sin\delta = \frac{\rho}{r}\sin\delta' - \frac{R}{r}\sin\delta_\Theta \quad (1.162)$$

The geocentric and barycentric coordinates are essentially the same for stars because /r 1 and R/r 0. For solar system objects, however, the coordinate computations must take the distances, r, and R into account. The logic is the same as that established for the celestial equatorial system if we use the ecliptic coordinate system. In terms of the geocentric ecliptic coordinates, point P's geocentric coordinates are:

$$x' = \rho\cos\beta'\cos\lambda' \quad (1.163)$$

$$y' = \rho\cos\beta'\sin\lambda' \quad (1.164)$$

$$z' = \rho\sin\beta' \quad (1.165)$$

The barycentric coordinates of point P in terms of geocentric ecliptic coordinates are:

$$x = r\cos\beta\cos\lambda \quad (1.166)$$

$$y = r\cos\beta\sin\lambda \quad (1.167)$$

$$z = r\sin\beta \quad (1.168)$$

The geocentric coordinates of the barycenter of the Solar System in terms of the ecliptic coordinates are:

$$X = R\cos\beta_\Theta \cos\lambda_\Theta \quad (1.169)$$
$$Y = R\cos\beta_\Theta \sin\lambda_\Theta \quad (1.170)$$
$$Z = R\sin\beta_\Theta \quad (1.171)$$

Thus, substituting Eq. (1.137), Eq. (1.138), and Eq. (1.139) into Eq. (1.131), the barycentric ecliptic coordinates of point P can be obtained from the equations:

$$\cos\beta\cos\lambda = \frac{\rho}{r}\cos\beta'\cos\lambda' - \frac{R}{r}\cos\beta_\Theta\cos\lambda_\Theta \quad (1.172)$$
$$\cos\beta\sin\lambda = \frac{\rho}{r}\cos\beta'\sin\lambda' - \frac{R}{r}\cos\beta_\Theta\sin\lambda_\Theta \quad (1.173)$$
$$\sin\beta = \frac{\rho}{r}\sin\beta' - \frac{R}{r}\sin\beta_\Theta \quad (1.174)$$

where are the barycentric ecliptic coordinates of point are the geocentric ecliptic coordinates of point are the coordinates of the solar system's barycenter. We may accept = 0 if it is within the necessary precision since the ecliptic latitude of the barycenter is often rather modest (100). Since the ecliptic system is used to analyze objects in the Solar System, we are unable to further simplify our analysis by taking into account the objects' distances.

CONCLUSION

The function of coordinate chirality in chemistry, which controls the characteristics and behavior of molecules. In order to comprehend how mirror-image compounds (enantiomers) may have significantly different effects in biological systems and pharmacology, chirality is crucial to the study of stereochemistry. We studied the manifestation of chirality in particle physics, where basic particles interact chirally and contribute to the structure of the world. In condensed matter physics, chirality also has a significant impact on how materials with distinct electrical characteristics behave. We also went into biology, realizing that chirality is a feature of life itself, from the double helix structure of DNA to the complex forms of proteins. This chirality is an essential component of the biological processes that support live creatures, not merely an abstract idea. We ask readers to reflect on the concept's enormous influence on how we see the world as we come to a conclusion on our examination of coordinate chirality. Chirality enhances our understanding of symmetry and asymmetry by revealing ideas that apply to both the large world of particle physics and the tiny world of molecules. By recognizing the significance of chirality, we create new opportunities for research, technological advancement, and a greater understanding of the complexity of the natural world.

REFERENCES:

[1] P. Satir, "Chirality of the cytoskeleton in the origins of cellular asymmetry," *Philos. Trans. R. Soc. B Biol. Sci.*, 2016, doi: 10.1098/rstb.2015.0408.

[2] I. Gamba *et al.*, "The folding of a metallopeptide," *Dalt. Trans.*, 2016, doi: 10.1039/c5dt02797g.

[3] J. Cerezo, G. Mazzeo, G. Longhi, S. Abbate, and F. Santoro, "Quantum-Classical Calculation of Vibronic Spectra along a Reaction Path: The Case of the ECD of Easily Interconvertible Conformers with Opposite Chiral Responses," *J. Phys. Chem. Lett.*, 2016, doi: 10.1021/acs.jpclett.6b02484.

[4] W. Liu *et al.*, “Nanowire Magnetoscope Reveals a Cellular Torque with Left-Right Bias,” *ACS Nano*, 2016, doi: 10.1021/acsnano.6b01142.

[5] K. Conley, L. Godbout, M. A. Whitehead, and T. G. M. Van De Ven, “Origin of the twist of cellulosic materials,” *Carbohydr. Polym.*, 2016, doi: 10.1016/j.carbpol.2015.08.029.

[6] H. F. Chen and W. R. Zhong, “Dynamics of chiral molecules in gaseous environments: Validity of the Magnus effect in microscale systems,” *RSC Adv.*, 2016, doi: 10.1039/c6ra02102f.

[7] Q. Bramas and S. Tixeuil, “Brief announcement: Probabilistic asynchronous arbitrary pattern formation,” in *Proceedings of the Annual ACM Symposium on Principles of Distributed Computing*, 2016. doi: 10.1145/2933057.2933074.

[8] P. Zhang, M. S. Zhu, H. Luo, Z. Li, and Y. B. Jiang, “Helical Aggregates of Building Blocks Formed In Situ from Five Components,” *Chempluschem*, 2016, doi: 10.1002/cplu.201600419.

[9] Q. Bramas and S. Tixeuil, “Probabilistic asynchronous arbitrary pattern pattern formation,” in *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 2016. doi: 10.1007/978-3-319-49259-9_7.

[10] D. Dong *et al.*, “Synthesis, structure, and electrochemistry and magnetic properties of a novel 1D homochiral MnIII(5-Brsalen) coordination polymer with left-handed helical character,” *J. Mol. Struct.*, 2016, doi: 10.1016/j.molstruc.2015.09.031.

CHAPTER 6

ANALYZING THE CONCEPT OF PLANE AND SPHERICAL TRIGONOMETRY

Kamesh Kumar, Assistant Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- drkamesh.engineering@tmu.ac.in

ABSTRACT:

Plane and Spherical Trigonometry is a foundational branch of mathematics that explores the relationships between angles and sides in triangles. In this comprehensive chapter, we embark on a journey to uncover the principles, formulas, and applications of both plane and spherical trigonometry. We begin by unraveling the essentials of plane trigonometry, delving into the trigonometric ratios, identities, and solving triangles.

As we traverse the world of spherical trigonometry, we transition to the intricacies of triangles on the surface of a sphere, unearthing the specialized formulas and principles that guide celestial navigation, astronomy, geodesy, and more. By the chapter's conclusion, readers will have acquired a solid foundation in both plane and spherical trigonometry, empowering them to tackle a wide range of mathematical and practical challenges we embarked on a comprehensive exploration of Plane and Spherical Trigonometry, two indispensable branches of mathematics with wide-ranging applications.

KEYWORDS:

Angles, Cosine, Geometry, Mathematics, Ratios, Sides.

INTRODUCTION

An overview of a planar triangle's solution. Although much of this will be known to readers, it is advised that it not be completely disregarded since the examples within provide some warnings about unnoticed risks [1], [2].

Triangles on a plane

To serve as a quick refresher on how to solve a planar triangle, read this section. Though it can be tempting to skim this part, it contains a caution that will become even more important in the section on spherical triangles. As seen in Figure 1, a plane triangle is typically defined by its three sides a, b, and c, which are opposed to each other and the three angles A, B, and C.

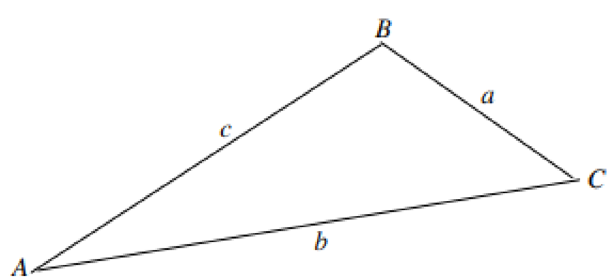


Figure 1: Illustrates the three component of plane triangle.

It is assumed that the reader is familiar with the sine and cosine formulas for the solution of the triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad 3.2.1$$

$$a^2 = b^2 + c^2 - 2bc\cos A, \qquad 3.2.2$$

It is aware that recognizing which formula is applicable under which conditions in Figure 2 is part of the art of solving a triangle. It will do to provide only two brief instances, each with a caution. Example: A plane triangle has sides of 7 inches by 4 inches by 28 degrees, and its angle B. Identify angle A [3], [4].

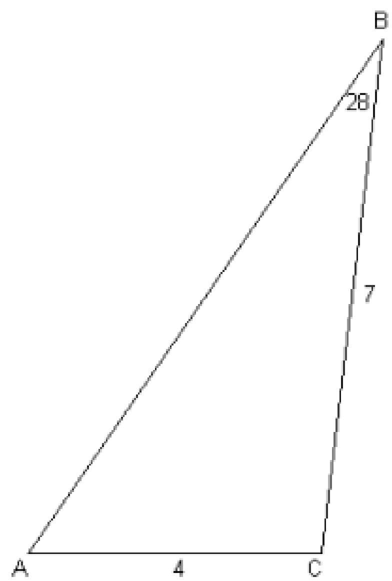


Figure 2: illustrates the angle of hypo-tense for plane triangle.

We use the sine formula, to obtain,

$$\sin A = \frac{7\sin 28^{\circ}}{4} = 0.821575$$

$$A = 55^{\circ}\ 14'.6$$

The trap is that, for two values of A between 0 and 180, only 55o 14'.6 and 124o 45'.4 fulfill sin A = 0.821575. Figure 3 demonstrates that any of them is a viable answer given the initial data.

All inverse trigonometric functions (sin-1, cos-1, tan-1) have two solutions between 0o and 360o, which is the lesson to be learnt from this. The function sin-1 is especially challenging since it has two solutions between 0o and 180o for positive parameters. It is advised that, unless special care is taken when programming calculators or computers, the reader should always be on the lookout for "quadrant problems" (i.e., figuring out which quadrant the desired solution belongs to). Quadrant problems are among the most frequent issues in trigonometry, and especially in spherical astronomy [5], [6].

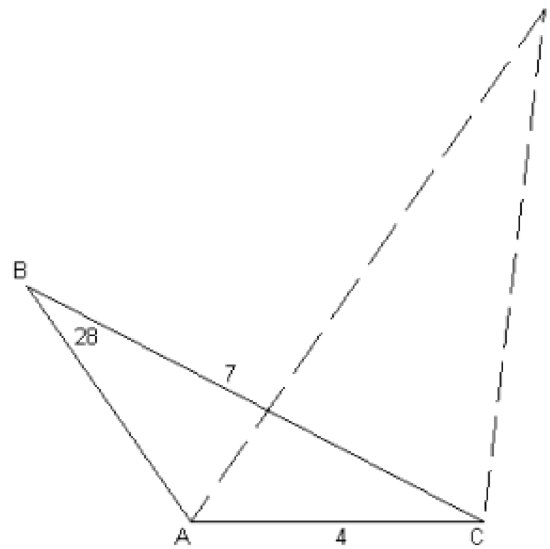


Figure 3: Illustrates to calculate the two value A and B.

Application of the cosine rule results in,

$$25 = x^2 + 64 - 16x \cos 32^\circ$$

Solution of the quadratic equation yields

x = 4.133 or 9.435

This demonstrates that the "two solutions" dilemma is not limited to angles. One of the solutions is shown to scale; the reader should depict the second alternative to demonstrate how two solutions are feasible.

The reader is now encouraged to use a hand calculator to attempt the following "guaranteed all different" tasks. Others could have two viable options. Some people may not. The reader must precisely depict the triangles, particularly those with two or no solutions. It's crucial to build a solid geometric grasp of trigonometric issues rather than just relying on a machine's automated computations.

The more difficult real-world issues faced in celestial mechanics and orbital computing will benefit from the development of these crucial abilities today.

PROBLEMS

1. a = 6 b = 4 c = 7 C =?

2. a = 5 b = 3 C = 43o c =?

3. a = 7 b = 9 C = 110o B =?

4. a = 4 b = 5 A = 29o c =?

5. a = 5 b = 7 A = 37o B =?

6. a = 8 b = 5 A = 54o C =?

7. A = 64o B = 37o a/c =? b/c =?

8. a = 3 b = 8 c = 4 C =?

9. a = 4 b = 11 A = 26 c =?

The reader is now further encouraged to create a computer program (in whatever language they are most comfortable with) that will solve each of the aforementioned issues for any value of the input data.

Lengths should be read in the input and printed to four significant figures in the output. Angles should be entered in degrees, minutes, and tenths of a minute (for example, 47 12'.9). output must appear.

Solutions to problems.

1. $C = 86^\circ\ 25'.0$
2. $c = 3.473$
3. $B = 40^\circ\ 00'.1$
4. $c = 7.555$ or 1.191
5. $B = 57^\circ\ 24'.6$ or $122^\circ\ 35'.4$
6. $C = 95^\circ\ 37'.6$
7. $a/c = 0.9165$ $b/c = 0.6131$
8. No real solution
9. No real solution

The area of a plane triangle is $\frac{1}{2}\times$ base $\times$ height, and it is easy to see from this that

$$\text{Area} \ = \tfrac{1}{2}bc\sin A = \tfrac{1}{2}ca\sin B = \tfrac{1}{2}ab\sin C \qquad 3.2.3$$

By making use $\sin^2 A = 1 - \cos^2 A$ and $\cos A = \left(b^2 + c^2 - a^2\right)/(2bc)$, we can express this entirely in terms of the lengths of the sides:

$$\text{Area} \ = \sqrt{s(s-a)(s-b)(s-c)}, \qquad 3.2.4$$

where s is the semi-perimeter $\frac{1}{2}(a+b+c)$.

Cylindrical and Spherical Coordinates

It is assumed that the reader is at least somewhat familiar with cylindrical coordinates (ρ, ϕ, z) and spherical coordinates (r, θ, ϕ) in three dimensions, and I offer only a brief summary here. Figure III.5 illustrates the following relations between them and the rectangular coordinates (x, y, z).

$$x = \rho\cos\phi = r\sin\theta\cos\phi \qquad 3.3.1$$

$$y = \rho\sin\phi = r\sin\theta\sin\phi \qquad 3.3.2$$

$$z = r\cos\theta \qquad 3.3.3$$

relations between spherical and rectangular coordinates are,

$$r = \sqrt{x^2 + y^2 + z^2} \qquad 3.3.4$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad 3.3.5$$

$$\phi = \cos^{-1} \frac{x}{\sqrt{x^2 + y^2}} = \sin^{-1} \frac{y}{\sqrt{x^2 + y^2}} \qquad 3.3.6$$

The terms "radial", "polar" or "meridional", and "azimuthal" are used to refer to the coordinates r,, and, respectively. R is fundamentally positive; the symbol indicates the square root's positive or absolute value. There is no quadrant uncertainty in the assessment of since the angle must be between 0 and 180. However, the angle may range from 0° to 360°. As a result, either the signs of x and y must be examined or both of the aforementioned formulae for must be assessed in order to determine uniquely. Calculating only based on = tan-1 (y/x) is insufficient. The reader should be aware, too, that certain calculator operations and computer languages will check the signs of x and y for you and return in the proper quadrant. For instance, in FORTRAN, if the variables X and Y are entered with the proper signs, the function ATAN2(X, Y) (or DATAN2(X,Y) in double precision) would return uniquely in the correct quadrant (albeit perhaps as a negative angle, in which case 360o should be added to the outputted angle). The reader should get acquainted with this function since it can save them a ton of time while programming [7], [8].

Cosines of direction

The two angles and may be used to define the direction to a point in three-dimensional space with respect to the origin. Giving the angles that the vector makes with the x-, y-, and z-axes, respectively, is another approach to describe the direction to a point or the orientation of a vector.

The cosines of these three angles are quoted more often. These are referred to as the direction cosines and are often represented as (l, m, n). The reader should be persuaded that the direction cosines and the angles and are related quite quickly:

$$l = \cos\alpha = \sin\theta\cos\phi \qquad 3.3.7$$

$$m = \cos\beta = \sin\theta\sin\phi \qquad 3.3.8$$

$$n = \cos\gamma = \cos\theta \qquad 3.3.9$$

These are not independent, and are related by

$$l^2 + m^2 + n^2 = 1. \qquad 3.3.10$$

A set of numbers that are multiples of the direction cosines - i.e. are proportional to them - are called *direction ratios*.

Latitude and Longitude.

The Earth's poles have a small flattening, which prevents it from being completely spherical. However, because our current goal is to familiarize ourselves with spherical coordinates and sphere geometry, we will assume that the Earth is spherical. In that scenario, the latitude, which is measured north or south of the equator, and the longitude, which is measured east or west from the meridian via Greenwich, may be used to represent the location of any town on Earth. Unfortunately, but often employed in this context, are the latitude and longitude symbols. The east longitude would be represented by and the 90o latitude by the symbols, for spherical coordinates that we have previously used [9], [10].

A sphere is intersected by a plane in a circle. The circle is referred to as a great circle if that plane crosses through the center of the sphere (making the center of the circle coincide with the center of the sphere). The equator and all meridians, which are fixed longitude circles that pass through the north and south poles, are large circles. This includes the one that goes through Greenwich. Small circles are formed by planes (such as parallels of latitude) that do not traverse the sphere's center. The radius of a latitude parallel is determined by multiplying the sphere's radius by the latitude's cosine.

In order to clarify the ideas of big and tiny circles, we utilized the example of latitude and longitude on a spherical Earth. We remark in passing that the true of the Earth at mean sea level is a geoid, which simply refers to the shape of the Earth, even if it is not necessary to follow it in the current context. With semi major axis a = 6378.140 km and semi minor axis c = 6356.755 km, the geoid is roughly an oblate spheroid (i.e., an ellipse rotated around its minor axis). The geometric ellipticity of the Earth is measured by the ratio (ac)/a, which has a value of 1/298.3.

The mean radius of the Earth is a c 3 2 = 6371.00 km, or the radius of a sphere with the same volume as the real geoid. A location on the surface of the Earth has a geographic or geodetic latitude, which must be distinguished from its geocentric latitude in accurate geodesy. They are defined clearly in Figure 4. The Earth's ellipticity is considerably overstated; in actuality, it would hardly be noticeable.

The angle is the angle formed between the equator and a plumb bob. This is different from'in part because a spheroid's gravitational field is different from that of a point mass at its center, and in part because the plumb bob is being dragged away from the Earth's rotation axis by centrifugal force.

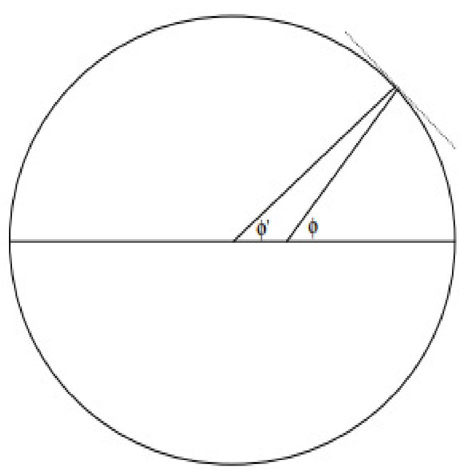


Figure 4: Illustrates thedistinguish between the geographic or geodetic latitude.

The relationship between φ and φ' is,

$$\phi - \phi' = 692'.74 \sin 2\phi - 1'.16 \sin 4\phi.$$

Velocity and Acceleration Components.

Two-dimensional polar coordinates

For two-dimensional polar coordinates, the symbols r and are sometimes used, however in this section I use to maintain consistency with the of three-dimensional spherical coordinates. I have bolded the vectors in the text that follows. You should be aware that certain printers do not seem to print Greek letter symbols in boldface, despite the fact that they show in boldface on screen, if you plan to print anything. This is something you should watch out for. Even if your printer does not understand that symbols with a above them are supposed to be unit vectors, you will still realize that they need to be in boldface. If in doubt, have a look at Figure 5 on the screen.

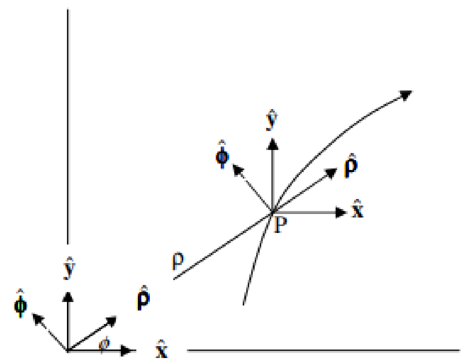


Figure 5: Illustrates thetwo-dimensional polar coordinates.

A point P that is travelling along a curve at a pace that causes its polar coordinates to change. φ & along with unit vectors in the radial and transverse directions, the picture also includes fixed unit vectors x and y that are parallel to the x- and y-axes. We'll look for expressions that describe how quickly the unit radial and transverse vectors change over time. (Being unit vectors, they don't vary in magnitude, but they do in direction.)

We have $$\hat{\boldsymbol{\rho}} = \cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}} \qquad 3.4.1$$

and $$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}. \qquad 3.4.2$$

∴ $$\dot{\hat{\boldsymbol{\rho}}} = -\sin\phi\,\dot{\phi}\,\hat{\mathbf{x}} + \cos\phi\,\dot{\phi}\,\hat{\mathbf{y}} = \dot{\phi}(-\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}) \qquad 3.4.3$$

∴ $$\dot{\hat{\boldsymbol{\rho}}} = \dot{\phi}\hat{\boldsymbol{\phi}}. \qquad 3.4.4$$

In a similar manner, by differentiating equation 3.4.2. with respect to time and then making use of equation 3.4.1, we find,

$$\dot{\hat{\boldsymbol{\phi}}} = -\dot{\phi}\,\hat{\boldsymbol{\rho}}$$

The rate of change of the radial and transverse unit vectors is given in equations 3.4.4 and 3.4.5. It is important to consider the implications of these two equations carefully. The equation = may be used to denote the position vector of the point P. By separating this with regard to time, one may get the velocity of P:

$$\mathbf{v} = \dot{\boldsymbol{\rho}} = \dot{\rho}\hat{\boldsymbol{\rho}} + \rho\dot{\hat{\boldsymbol{\rho}}} = \dot{\rho}\hat{\boldsymbol{\rho}} + \rho\dot{\phi}\hat{\boldsymbol{\phi}}.$$

Therefore, & and &, respectively, represent the radial and transverse components of velocity. Equation 3.4.6's differentiation is used to get the acceleration, and we must also differentiate the products of two and three time-varying components:

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\rho}\hat{\boldsymbol{\rho}} + \dot{\rho}\dot{\hat{\boldsymbol{\rho}}} + \dot{\rho}\dot{\phi}\hat{\boldsymbol{\phi}} + \rho\ddot{\phi}\hat{\boldsymbol{\phi}} + \rho\dot{\phi}\dot{\hat{\boldsymbol{\phi}}}$$

$$= \ddot{\rho}\hat{\boldsymbol{\rho}} + \dot{\rho}\dot{\phi}\hat{\boldsymbol{\phi}} + \dot{\rho}\dot{\phi}\hat{\boldsymbol{\phi}} + \rho\ddot{\phi}\hat{\boldsymbol{\phi}} - \rho\dot{\phi}^2\hat{\boldsymbol{\rho}}$$

$$= \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\boldsymbol{\rho}} + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\hat{\boldsymbol{\phi}}.$$

Figure 6 shows the radial and transverse components of acceleration.

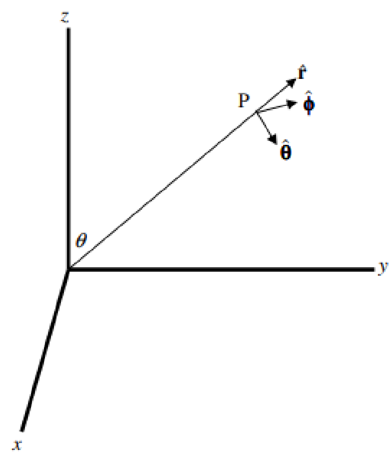


Figure 6: Illustrates the radial and transverse components of acceleration in plane.

In P is a point moving along a curve such that its spherical coordinates are changing at rates θ φ. We want to find out how fast the unit vectors θ φˆ, ˆ rˆ, in the radial, meridional and azimuthal directions are changing.

We have $\hat{\mathbf{r}} = \sin\theta\,\cos\phi\,\hat{\mathbf{x}} + \sin\theta\,\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$ 3.4.8

$$\hat{\boldsymbol{\theta}} = \cos\theta\,\cos\phi\,\hat{\mathbf{x}} + \cos\theta\,\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \qquad 3.4.9$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \qquad 3.4.10$$

$$\therefore \quad \dot{\hat{\mathbf{r}}} = (\cos\theta\,\dot{\theta}\cos\phi - \sin\theta\sin\phi\,\dot{\phi})\hat{\mathbf{x}} + (\cos\theta\,\dot{\theta}\sin\phi + \sin\theta\cos\phi\,\dot{\phi})\hat{\mathbf{y}} - \sin\theta\,\dot{\theta}\,\hat{\mathbf{z}}. \qquad 3.4.11$$

We see, by comparing this with equations 3.4.9 and 3.4.10 that

$$\dot{\hat{\mathbf{r}}} = \dot{\theta}\,\hat{\boldsymbol{\theta}} + \sin\theta\,\dot{\phi}\,\hat{\boldsymbol{\phi}} \qquad 3.4.12$$

By similar arguments we find that

$$\dot{\hat{\boldsymbol{\theta}}} = \cos\theta\,\dot{\phi}\,\hat{\boldsymbol{\phi}} - \dot{\theta}\hat{\mathbf{r}} \qquad 3.4.13$$

and $$\dot{\hat{\boldsymbol{\phi}}} = -\sin\theta\,\dot{\phi}\,\hat{\mathbf{r}} - \cos\theta\,\dot{\phi}\,\hat{\boldsymbol{\theta}} \qquad 3.4.14$$

These are the radial, meridional, and azimuthal vectors' rates of change. The equation r = r r may be used to denote the position vector of the point P. By separating this with regard to time, one may get the velocity of P:

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\,\hat{\mathbf{r}} + r\,\dot{\hat{\mathbf{r}}} = \dot{r}\,\hat{\mathbf{r}} + r(\dot{\theta}\,\hat{\boldsymbol{\theta}} + \sin\theta\,\dot{\phi}\,\hat{\boldsymbol{\phi}})$$

$$= \dot{r}\,\hat{\mathbf{r}} + r\,\dot{\theta}\,\hat{\boldsymbol{\theta}} + r\sin\theta\,\dot{\phi}\,\hat{\boldsymbol{\phi}} \qquad 3.4.15$$

Therefore, r& r &, r &, and r sin & are the radial, meridional, and azimuthal components of velocity, respectively. Equation 3.4.15 is differentiated to get the acceleration. [Perhaps a little differentiating tip would be appropriate here. The majority of readers are likely to be able to tell a product of two functions apart. The formula is (abcd)' = a'bcd + ab'cd + abc'd + abcd' if you wish to differentiate a product of numerous functions, for instance four functions, a, b, c, and d. All four values in the last part of equation 3.4.15 fluctuate with time, and we are going to distinguish the product.]

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{r}\,\hat{\mathbf{r}} + \dot{r}(\dot{\theta}\,\hat{\boldsymbol{\theta}} + \sin\theta\,\dot{\phi}\,\hat{\boldsymbol{\phi}}) + \dot{r}\dot{\theta}\,\hat{\boldsymbol{\theta}} + r\ddot{\theta}\,\hat{\boldsymbol{\theta}} + r\dot{\theta}(\cos\theta\,\dot{\phi}\,\hat{\boldsymbol{\phi}} - \dot{\theta}\hat{\mathbf{r}})$$

$$+ \dot{r}\sin\theta\,\dot{\phi}\,\hat{\boldsymbol{\phi}} + r\cos\theta\,\dot{\theta}\dot{\phi}\,\hat{\boldsymbol{\phi}} + r\sin\theta\,\ddot{\phi}\,\hat{\boldsymbol{\phi}} + r\sin\theta\,\dot{\phi}(-\sin\theta\,\dot{\phi}\,\hat{\mathbf{r}} - \cos\theta\,\dot{\phi}\,\hat{\boldsymbol{\theta}}) \qquad 3.4.16$$

On gathering together the coefficients of $\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}$, we find that the components of acceleration are:

Radial: $\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\,\dot{\phi}^2$

Meridional: $r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\,\dot{\phi}^2$

Azimuthal: $2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + r\sin\theta\,\ddot{\phi}$

CONCLUSION

We started our adventure by laying the foundation for comprehending triangle angles and sides in the field of plane trigonometry. We presented the basic trigonometric ratios—sine, cosine, and tangent—and showed how they may be used to address a number of right triangle-related issues. We looked at trigonometric identities, which offered useful instruments for reducing the complexity of complicated expressions and equations. Readers had mastered the skill of solving triangles by the conclusion of the plane trigonometry section, accurately computing side lengths, angles, and areas. We next switched to spherical trigonometry and entered the fascinating realm of triangles on a sphere's surface. We outlined the fundamental ideas of spherical coordinates while emphasizing their importance in disciplines like astronomy, navigation, and geodesy. The chapter clarified the special characteristics of spherical triangles, highlighting the idea of spherical excess—a crucial distinction from their planar equivalents. We explored unique spherical trigonometric identities and formulae, such the Law of Sines and Law of Cosines for spherical triangles, and presented useful tools like the Haversine formula for computing great-circle distances on Earth. These formulae were used in celestial navigation, where spherical trigonometry's concepts were crucial for figuring out celestial locations, distances, and bearings.

REFERENCES:

[1] E. Emmanouil, G. Wei, and J. S. Dai, "Spherical trigonometry constrained kinematics for a dexterous robotic hand with an articulated palm," *Robotica*, 2016, doi: 10.1017/S0263574715000399.

[2] J. Conway and A. Ryba, "Remembering spherical trigonometry," *Math. Gaz.*, 2016, doi: 10.1017/mag.2016.3.

[3] C. L. Chen, "A systematic approach for solving the great circle track problems based on vector algebra," *Polish Marit. Res.*, 2016, doi: 10.1515/pomr-2016-0014.

[4] W. Zhang, X. Xu, and Y. Wu, "A new method of single celestial-body sun positioning based on theory of mechanisms," *Chinese J. Aeronaut.*, 2016, doi: 10.1016/j.cja.2015.12.009.

[5] M. Bagheri, "Kushyar IBN Labban's mathematical approach in his astronomical handbook," *Micrologus*. 2016.

[6] S. Y. Ham, S. H. Kim, and Y. S. Lee, "A study on making a Honsang using the star catalogue from [Seong Gyeong]," *Adv. Sp. Res.*, 2016, doi: 10.1016/j.asr.2015.10.042.

[7] S. Raghavan, "Cakravala Method," in *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, 2016. doi: 10.1007/978-94-007-7747-7_9221.

[8] J. L. Berggren, "Spherical Trigonometry in the Islamic World," in *Episodes in the Mathematics of Medieval Islam*, 2016. doi: 10.1007/978-1-4939-3780-6_6.

[9] M.-J. Jeon, E. Kim, S.-B. Lim, and S.-W. Choi, "A line rate calculation method for arbitrary directional imaging of an Earth observing satellite," in *Sensors, Systems, and Next-Generation Satellites XX*, 2016. doi: 10.1117/12.2241962.

[10] J. A. Favorite, "The solid angle for a spherical surface source and a detector with an arbitrary aperture: An elegant proof," in *Transactions of the American Nuclear Society*, 2016.

CHAPTER 7

GEOMETRY AND GEOGRAPHY OF SPHERICAL TRIANGLES

Vijendra Singh Rawat, Assistant Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- vijendr.rawat@gmail.com

ABSTRACT:

Spherical Triangles represent a captivating intersection of geometry and geography, essential for understanding and navigating the curved surfaces of our planet and celestial bodies. In this illuminating chapter, we embark on a journey into the intricate world of spherical triangles, exploring their unique properties, specialized formulas, and practical applications. We begin by introducing the fundamental concepts, such as the vertices, sides, and angles of spherical triangles.

As we delve deeper, we unravel the mathematical tools, including the Law of Sines and the Law of Cosines for spherical triangles, which are indispensable for geodesy, celestial navigation, and astronomy. By the chapter's culmination, readers will have acquired a profound comprehension of spherical triangles and their pivotal role in exploring and measuring the spherical wonders of our universe we have ventured into the captivating realm of spherical triangles, uncovering the principles, formulas, and applications that make them an essential facet of both terrestrial and celestial geometry. Our journey began with the fundamental concepts of spherical triangles—the vertices, sides, and angles that define these unique geometric figures.

Unlike planar triangles, spherical triangles exist on the curved surface of a sphere, where traditional Euclidean geometry gives way to the intricacies of spherical geometry.

KEYWORDS:

Angles, Geometry, Geography, Spherical, Spherical Triangles, Triangles.

INTRODUCTION

Like with plane triangles, we use the letters A, B, and C to identify the three angles and the corresponding sides. The skill of solving a spherical triangle is knowing which formula to use depending on the situation.

We are lucky in that we have four formulae at our disposal for solving a spherical triangle. Each formula has four components (sides and angles), three of which are presumed to be known in a particular issue and the fourth of which has to be calculated [1], [2].Before we write down the formulae, there are three crucial things to remember.

1. Only triangles with three sides that are arcs of great circles are suitable for the formulae. They won't work, for instance, in a triangle if one side is a latitude parallel.

2. Rather of being described in linear units like meters or kilometers, the sides and angles of a spherical triangle are all expressed in angular units like degrees and minutes. A side of 50° denotes an arc of a large circle extending at an angle of 50° from the sphere's center.

3. A spherical triangle's three angles add up to more than 180 degrees.

The four formulae are now presented in this part without justification; the derivations are provided in a subsequent section. The sine formula, the cosine formula, the polar cosine formula, and the cotangent formula are the four equations. Each formula is shown with a spherical triangle below it, with the four components of the formula underlined [3], [4].

The sine formula:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}\left(= \frac{\sin c}{\sin C}\right)$$

The cosine formula:

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

The cotangent formula:

$$\cos b \cos A = \sin b \cot c - \sin A \cot C$$

It is unfortunate that, even with the formula written out in front of one, it is usually challenging to determine which is b, which is A, and so on. The cotangent formula is a very valuable and frequently required formula. However, it should be noticed from the diagram that the triangle's four constituent parts—side, angle, angle, and side—lie close to one another. Accordingly, they may be referred to as the triangle's outer side (OS), inner angle (IA), inner side (IS), and outer angle (OA). Many individuals find that writing the formula in the form makes it much simpler to utilize:

$$\cos(\mathrm{IS})\cos(\mathrm{IA}) = \sin(\mathrm{IS})\cot(\mathrm{OS}) - \sin(\mathrm{IA})\cot(\mathrm{OA}) \qquad 3.5.5$$

Soon, the reader will be given a sizable number of examples on how to utilize these formulae. However, when utilizing the formulae, it will become apparent that solving deceptively simple trigonometric equations of the type,

$$4.737\sin\theta + 3.286\cos\theta = 5.296$$

One of numerous approaches may then be presented to the reader after a little wait, albeit not all are equally effective. I'll provide four potential solutions to this equation. The reader may immediately consider the first way to be pretty clear, but there is a cautionary story associated with it.

The reader would be encouraged to choose one of the less apparent approaches since even if the method may seem to be extremely straightforward, a complication does occur. The equation between 0 and 360 has two solutions, by the way [5], [6].

They are 31o 58'.6 and 78o 31'.5.

i. Method

The obvious method is to isolate cos θ:

$$\cos\ \theta\ = 1.611\,686 - 1.441\,570\sin\theta.$$

Do not be tempted to round off intermediate computations to four despite the fact that the constants in the problem were supplied to four significant digits. Prematurely rounding off intermediate computations is a frequent mistake. At the conclusion, the rounding-off may be done. Square both sides, and write the left hand side, cos 2 θ , as 1 − sin2 θ . We now have a quadratic equation in sin θ:

$$3.078\,125\sin^2\theta - 4.646\,717\sin\theta + 1.597\,532 = 0.$$

The four values of that fulfill these values of sin are: 31o 58'.6, 148o 01'.4, 78o 31'.5, and 101o 28'.5. The two solutions for sin are: 0.529 579 and 0.908 014.

There are only two of these angles that are answers to the initial equation. Squaring both sides of the original equation was the fatal error, and as a result, we have discovered solutions not only to [7], [8].

$$\cos\theta = 1.611\,686 - 1.441\,570 \sin\theta$$

but also, to the different equation,

$$-\cos\theta = 1.611\,686 - 1.441\,570 \sin\theta.$$

Every time we square an equation, more solutions are generated.

Because of this, technique (i), although alluring, ought to be avoided, especially when programming a computer to do an operation automatically and without thought. whether you're unsure whether you've found the right answer, enter your answer in place of the original equation. This is something you should always perform with any kind of equation.

ii. Method

This method makes use of the identities:

$$\sin\theta = \frac{2t}{1+t^2}, \qquad \cos\theta = \frac{1-t^2}{1+t^2},$$

where t = tan. 1 2 θ When applied to the original equation, this results in the quadratic equation in t:

$$8.582t^2 - 9.474t + 2.010 = 0$$

$$t = 0.286528 \quad \text{and} \quad t = 0.817410.$$

The only values of θ between 0o and 360o that satisfy these are the two correct solutions 31 o 58'.6 and 78o 31'.5. It is left as an exercise to show, using this method algebraically, that the solutions to the equation,

$$a\sin\theta + b\cos\theta = c$$

$$\tan\tfrac{1}{2}\theta = \frac{a \pm \sqrt{a^2+b^2-c^2}}{b+c}.$$

This shows that there are no real solutions if $a^2 + b^2 < c^2$, one real solution if $a^2 + b^2 = c^2$, and two real solutions if $a^2 + b^2 > c^2$.

Method iii.

We divide the original equation

$$4.737 \sin\theta + 3.286 \cos\theta = 5.296$$

by the "hypotenuse" of 4.737 and 3.286; that is, by $\sqrt{(4.737^2 + 3.286^2)} = 5.765151$.

Thus $$0.821\,661 \sin\theta + 0.569\,976 \cos\theta = 0.918\,623$$

Now let 0.821 661 = cos α and 0.569 976 = sin α (which we can, since these numbers now satisfy $\sin^2\alpha + \cos^2\alpha = 1$) so that $\alpha = 34^\circ\,44'.91$.

We have $$\cos\alpha \sin\theta + \sin\alpha \cos\theta = 0.918\,623$$

or $$\sin(\theta+\alpha) = 0.918623$$

from which $$\theta + \alpha = 66^\circ\,43'.54 \quad \text{or} \quad 113^\circ\,16'.46$$

Therefore $$\theta = 31^\circ\,58'.6 \quad \text{or} \quad 78^\circ\,31'.5$$

iv. Method

There may not be a need to employ numerical approaches since procedures ii and iii provide clear solutions. However, the reader may find it interesting to solve the problem using Newton-Raphson iteration,

$$f(\theta) = a\sin\theta + b\cos\theta - c = 0,$$

$$f'(\theta) = a\cos\theta - b\sin\theta.$$

Using the values of a, b and c from the example above and using the Newton-Raphson algorithm, we find with a first guess of 45o the following iterations, working in radians:

0.785 398

0.417 841

0.541 499

0.557 797

0.558 104

0.558 104 = 31^0 58'.6

The reader should validate this computation and demonstrate that Newton-Raphson iteration gets to 78o 31'.5 rapidly using an alternative starting guess.

After overcoming that little obstacle, the reader is urged to address the spherical triangle issues listed below. These twelve issues seem to be meaningless repetitions; however, they are all unique. Between 0o and 360o, some have two answers, while others only have one. The reader should draw each triangle after answering each issue, particularly those with two answers, to illustrate how the two-fold ambiguities develop. The reader should also create a computer software that will execute the user's instructions to solve each of the twelve different sorts of problems. Answers must be accurate to those precisions in degrees, minutes, and tenths of a minute. For instance, 47o 37'.3 is the solution to one of the issues. 47o 37'.2 or

47o 37'.4 shall be considered as incorrect answers. Answers that are "nearly right" have no place in celestial mechanics. Either a response is correct or it is incorrect [9], [10]. (This does not imply that an angle can be measured accurately; but, the result of a calculation made to a precision of a tenth of an arcminute should be accurate to a tenth of an arcminute.)

PROBLEMS

(All angles and sides in degrees.)

10. a = 64 b = 33 c = 37 C =?

11. a = 39 b = 48 C = 74 c =?

12. a = 16 b = 37 C = 42 B =?

13. a = 21 b = 43 A = 29 c =?

14. a = 67 b = 54 A = 39 B =?

15. a = 49 b = 59 A = 14 C =?

16. A = 24 B = 72 c = 19 a =?

17. A = 79 B = 84 c = 12 C =?

18. A = 62 B = 49 a = 44 b =?

19. A = 59 B = 32 a = 62 c =?

20. A = 47 B = 57 a = 22 C =?

21. A = 79 B = 62 C = 48 c =?

Solutions to problems.

10. 28^{o} 18'.2
11. 49^{o} 32'.4
12. 117^{o} 31'.0
13. 30^{o} 46'.7 or 47^{o} 37'.3
14. 33^{o} 34'.8
15. 3^{o} 18'.1 or 162^{o} 03'.9
16. 7^{o} 38'.2
17. 20^{o}46'.6
18. 36^{o} 25'.5
19. 76^{o} 27'.7
20. 80^{o} 55'.7 or 169^{o} 05'.2
21. 28^{o} 54'.6

Derivation of the formulas

It is time to derive the four formulae that have just recently been supplied without supporting evidence before going on to more issues and applications of the formulas. With the cosine formula, we will begin. Choosing rectangular axes such that the point A of the spherical triangle ABC is on the z-axis and the point B and therefore the side care in the zx-plane does not reduce the generality of the solution. It is believed that the sphere has a radius of one. The position vectors of the points B and C with respect to the sphere's center are shown in the picture if i, j, and k are unit vectors oriented along the x, y, and z axes, respectively:

$$\mathbf{r_1} = \mathbf{i}\sin c + \mathbf{k}\cos c \qquad 3.5.7$$

and

$$\mathbf{r_2} = \mathbf{i}\sin b\cos A + \mathbf{j}\sin b\sin A + \mathbf{k}\cos b \qquad 3.5.8$$

respectively.

The cosine of the angle between two vectors, or cos a, is the only component of their scalar product (each of magnitude unity), from which we may instantly get,

$$\cos a = \cos b\cos c + \sin b\sin c\cos A.$$

To obtain the sine formula, we isolate cos A from this equation, square both sides, and write 1 2 – sin A for cos2 A. Thus,

$$(\sin b\sin c\cos A)^2 = (\cos a - \cos b\cos c)^2,$$

and when we have carried out these operations we obtain,

$$\sin^2 A = \frac{\sin^2 b\sin^2 c - \cos^2 a - \cos^2 b\cos^2 c + 2\cos a\cos b\cos c}{\sin^2 b\sin^2 c}.$$

In the numerator, write 1 2 – cos b for sin2 b and 1 2 – cos c for sin2 c, and divide both sides by sin2 a. This results in,

$$\frac{\sin^2 A}{\sin^2 a} = \frac{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2\cos a\cos b\cos c}{\sin^2 a\sin^2 b\sin^2 c}.. \qquad 3.5.12$$

A brief feeling of joy might likewise result from the polar cosine formula's derivation. A'B'C' is a spherical triangle in Figure 1. The polar triangle, often known as A'B'C, is a spherical triangle that includes ABC. It takes the following form. The side BC is an arc of a great circle that is 90 degrees away from the pole A', meaning that BC is a portion of the equator. CA and AB are 90 degrees apart from B' and C', respectively. The little triangle's side B'C' has been stretched in the illustration to meet the giant triangle's sides AB and CA. The illustration makes it clear that the huge triangle's angle A is equal to x + a' + y. Furthermore, x+ a' and a' + y is both equivalent to 90o based on how the triangle ABC was built. These connections show that,

$$A + A = [(x + a') + y] + [x + (a' + y)]$$

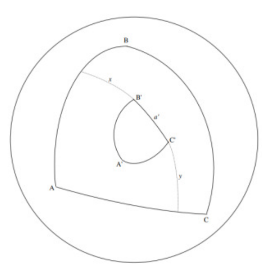


Figure 1: illustrates show that the derivation of the polar cosine formula may also bring a small moment of delight.

or $2A = 180^{\circ} + x + y = 180^{\circ} + A - a'$

Therefore $A = 180^{\circ} - a'$

In a similar manner, $B = 180^{\circ} - b'$ and $C = 180^{\circ} - c'$

Assume that every connection between the sides and angles of the triangle A'B'C' has the value f(A', B', C', a', b', c') = 0. A relation between A, B, C, a, b, and c, or a relation between the sides and angles of the triangle ABC, will arise if we substitute a' with 180o of A, b' with 180o of B, and so on.

For example, the equation,

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'$$

is valid for the triangle A'B'C'. By making these substitutions, we find the following formula valid for triangle ABC:

$$-\cos A = \cos B \cos C - \sin B \sin C \cos a,$$

which is the polar cosine formula.

The reader will probably attempt beginning with the sine and cotangent formulae for the triangle A'B'C' and figuring out comparable polar formulas for the triangle ABC, but this may, regrettably, lead to some rather unsatisfying results. The cotangent formula's derivation is not especially fascinating to me, so I'll leave it up to the reader to figure out the pretty simple algebra.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

and $$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

Eliminate $\cos c$ (but retain $\sin c$) from these equations, and write $1 - \sin^2 b$ for $\cos^2 b$. Finally substitute $\frac{\sin c \sin A}{\sin C}$ for $\sin a$, and, after some tidying up, the cotangent formula should result.

CONCLUSION

Spherical triangles are distinguished from their planar counterparts by the idea of spherical excess. The concept of spherical excess expresses the understanding that the total of the angles in a spherical triangle exceeds 180 degrees, which has significant ramifications for both celestial navigation and geodesy. We looked at the spherical trigonometry's specific formulae, such as the Law of Sines and Law of Cosines for spherical triangles. These formulae are very useful for calculating celestial locations, measuring distances on the surface of the Earth, and resolving challenging geometric issues in astronomy and geodesy. We ask readers to acknowledge the lasting importance of spherical triangles as we draw to a close. They serve as the basis for accurate measurements of distances and bearings on the curved surface of the Earth in the field of geodesy. They make it possible for astronomers and seafarers to accurately traverse the skies and the oceans. They serve as a powerful reminder of the intricate connections between mathematics, geography, and the glories of our spherical world thanks to their mathematical beauty and usefulness. Spherical triangle research is a demonstration of the elegance and complexity of the mathematical structures that underlie our comprehension of the universe, both on Earth and in space.

REFERENCES:

[1] E. Emmanouil, G. Wei, and J. S. Dai, "Spherical trigonometry constrained kinematics for a dexterous robotic hand with an articulated palm," *Robotica*, 2016, doi: 10.1017/S0263574715000399.

[2] C. L. Chen, "A systematic approach for solving the great circle track problems based on vector algebra," *Polish Marit. Res.*, 2016, doi: 10.1515/pomr-2016-0014.

[3] J. Conway and A. Ryba, "Remembering spherical trigonometry," *Math. Gaz.*, 2016, doi: 10.1017/mag.2016.3.

[4] L. Kowalski *et al.*, "Spherical Wind Sensor for the Atmosphere of Mars," *IEEE Sens. J.*, 2016, doi: 10.1109/JSEN.2015.2509168.

[5] M. Grzelczak *et al.*, "Silver Ions Direct Twin-Plane Formation during the Overgrowth of Single-Crystal Gold Nanoparticles," *ACS Omega*, 2016, doi: 10.1021/acsomega.6b00066.

[6] B. Alsadik, "A modified method for image triangulation using inclined angles," in *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences - ISPRS Archives*, 2016. doi: 10.5194/isprsarchives-XLI-B3-453-2016.

[7] A. Y. Dzyublik and V. Y. Spivak, "Laue diffraction of spherical Mössbauer waves," *Ukr. J. Phys.*, 2016, doi: 10.15407/ujpe61.09.0826.

[8] V. I. Travush, V. D. Antoshkin, and V. T. Erofeev, "The problem 4 of placement triangular geometric line field," in *MATEC Web of Conferences*, 2016. doi: 10.1051/matecconf/20168601031.

[9] S. Ghosh *et al.*, "Dioscorea oppositifolia Mediated Synthesis of Gold and Silver Nanoparticles with Catalytic Activity," *J. Nanomed. Nanotechnol.*, 2016, doi: 10.4172/2157-7439.1000398.

[10] C. Sun, Z. Guo, K. Wang, and N. Cheng, "Energy Optimized Parameterization of Spherical Triangle Mesh," 2016. doi: 10.2991/icca-16.2016.60.

CHAPTER 8

CONCEPT OF COORDINATES OF THE AZIMUTH ANGLE

Rajiv Verma, Professor
Department of Management, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- dr.rajiv_verma@rediffmail.com

ABSTRACT:

The azimuth angle, a fundamental concept in both astronomy and navigation, serves as a vital guidepost for determining the direction of celestial objects or terrestrial landmarks. In this comprehensive chapter, we delve into the coordinates of the azimuth angle, exploring its significance, measurement, and practical applications. We begin by defining azimuth as the angular measurement of an object's direction, relative to a reference point, often the north direction. Our journey takes us through the mathematical foundations of azimuth and its relation to horizontal coordinates. We explore the diverse range of applications, from celestial navigation to surveying and even the tracking of celestial events. By the chapter's end, readers will have gained a deep understanding of how the coordinates of the azimuth angle are a crucial tool for finding one's way on Earth and observing the celestial wonders above we've embarked on an exploration of the coordinates of the azimuth angle, uncovering its importance in both terrestrial and celestial contexts.

We began our journey by establishing azimuth as a fundamental concept, representing the angular measurement of an object's direction relative to a reference point.

Commonly, this reference point is the north direction, but it can be any chosen orientation. This angular measurement system plays a pivotal role in understanding the orientation and location of celestial objects, landmarks, or targets.

KEYWORDS:

Angle, Azimuth, Coordinates, Measurement, Navigation.

INTRODUCTION

At this point, we have obtained the four spherical triangle formulae and have practiced solving them. In this part, we come across cases where the goal is not only to solve a triangle but to learn how to frame a problem and choose which triangle has to be solved [1], [2].

1. The David Dunlap Observatory is located near Toronto, Ontario, at latitude 43o 51'.8 north, longitude 79o 25'.3 west, while the Dominion Astrophysical Observatory is located close to Victoria, British Columbia, at latitude 48o 31'.3 north, longitude 123o 25'.0 west.

PVT, where P is the north pole of the Earth, V is Victoria, and T is Toronto, is the triangle that has to be drawn and solved. The two cities' colatitudes and the difference between their longitudes are shown on Figure 1.

The great circle distance ω between the two observatories is easily given by the cosine formula:

$$\cos \omega = \cos 41^\circ\, 28'.7 \cos 46^\circ\, 08'.2 + \sin 41^\circ\, 28'.7 \sin 46^\circ\, 08'.2 \cos 43^\circ\, 59'.7$$

We calculate = 300 22'.7 or 0.53021 radians from this. The distance between the observatories is 3378 km (2099 miles) due to the Earth's 6371 km radius [3], [4].

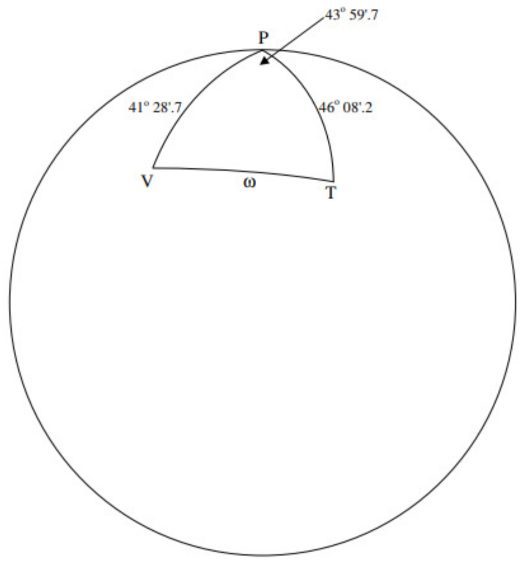


Figure 1: Illustrates show that the colatitudes of the two cities and the difference between their longitudes.

Now that we have found ω, we can find the azimuth, which is the angle V, from the sine formula:

$$\sin V = \frac{\sin 46^o 08'.2 \sin 43^o 59'.7}{\sin 30^o 22'.7} = 0.990\,275$$

and hence $V = 82^o\, 00'.3$

Sin-1 0.990 275, however, has two values between 0 and 180 degrees, namely 82o 00'.3 and 97o 59'.7. Usually, it is evident from examination of a drawing which of the two values of sin-1 is the needed one. Unfortunately, in this scenario, both numbers are near to 90o, and it may not be immediately evident which of the two values we need. To clear up any confusion, it should be noted that Toronto is located at a latitude that is farther southerly than Victoria [5], [6].

Of fact, we could have used the cotangent formula to get the azimuth V without first determining. Thus

cos 41o 28'.7 cos 43o59'.7 = sin 41o 28'.7 cot 46o 08'.2 sin 43o 59'.7 cot V

V can only have one solution between 0 and 180, and that solution—82o 00'—is the right one.3. A good drawing will explain to the viewer why the acute angle, rather than the obtuse angle, was the correct answer (in our illustration, the angle was made to be close to 900 to avoid favoring either side), but in any case, all viewers, especially those who were pressured into selecting the obtuse angle, should pay close attention to the challenges that can be brought on by the function sin's ambiguity. Despite how simple it is to memorize; the author strongly advises against ever using the sine formula. The cotangent formula is harder to remember, but it is far more accurate and less prone to quadrant errors.

Take into account two sites, A and B, located at latitudes 20° N and 25° E and 72° N and 44° E, respectively. Where do the great circle's poles meet at these two points? Three approaches to the issue will be presented. By resolving spherical triangles, start with (a). Second, utilizing the algebraic coordinate geometry techniques, as recommended by Achintya Pal. And third, (c), which J. Viswanathan proposed to me.

(a) Let us call the colatitude and longitude of the first point () 1 1 θ, φ and of the second point (,). θ2 φ2 We shall consider the question answered if we can find the coordinates () 0 0 θ, φ of the poles Q and Q' of the great circle passing through the two points. In Figure 2, P is the north pole of the Earth, A and B are the two points in question, and Q is one of the two poles of the great circle joining A and B. The figure also shows the triangle PQA. We'll suppose that the origin for longitudes ("Greenwich") is behind the plane of the paper. The east longitudes of Q, A and B are, respectively, 0 1 2 φ, φ, φ ; and their colatitudes are 0 1 2 θ , θ ,θ.

$$0 = \cos\theta_0 \cos\theta_1 + \sin\theta_0 \sin\theta_1 \cos(\phi_1 - \phi_0),$$

from which,

$$\tan\theta_0 = -\frac{1}{\tan\theta_1 \cos(\phi_1 - \phi_0)}.$$

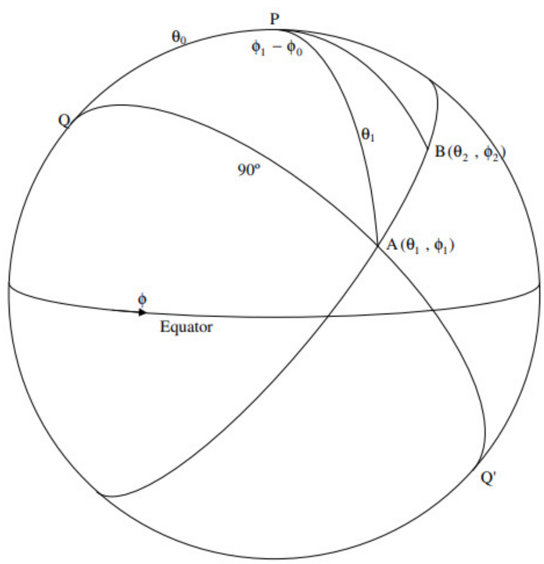


Figure 2: Shown that the Equator side of earth.

Similarly, from triangle PQB we would obtain,

$$\tan\theta_0 = -\frac{1}{\tan\theta_2 \cos(\phi_2 - \phi_0)}.$$

These are two equations in θ 0 and φ 0, so the problem is in principle solved. Equate the righthand sides of the two equations, expand the terms cos (φ φ) 1 − 0 and cos (φ φ), 2 − 0 gather the terms in sin φ 0 and cos φ 0 , eventually to obtain,

$$\tan\phi_0 = \frac{\tan\theta_1\cos\phi_1 - \tan\theta_2\cos\phi_2}{\tan\theta_2\sin\phi_2 - \tan\theta_1\sin\phi_1}.$$

If we substitute the angles given in the original problem, we obtain,

$$\tan\phi_0 = \frac{\tan 70^\circ \cos 25^\circ - \tan 18^\circ \cos 44^\circ}{\tan 18^\circ \sin 44^\circ - \tan 70^\circ \sin 25^\circ} = -2.412\,091\,0$$

from which,

$$\phi_0 = 112^\circ\ 31'.1 \quad \text{or} \quad 292^\circ\ 31'.1$$

We next use one of the tan 0 equations to get 0 (it is recommended to utilize both of them to double-check the math). There can be no uncertainty about the quadrant since the north polar distance, or colatitude, must be between 0o and 180 [7], [8].

With $\phi_0 = 112^\circ\ 31'.1$, we obtain $\theta_0 = 96^\circ\ 47'.1$, i.e. latitude $6^\circ\ 47'.1$ S.
and with $\phi_0 = 292^\circ\ 31'.1$, we obtain $\theta_0 = 83^\circ\ 12'.9$, i.e. latitude $6^\circ\ 47'.1$ N.

and these are the positions of the two great circle poles that cross through points A and B. To be absolutely certain that the quadrants are accurate and clear, the reader is highly encouraged to actually do these calculations numerically. In fact, solving the quadrant issue may be seen of as the exercise's most crucial component.

Pal's method

Equations 3.5.17 and 3.5.18 were obtained by using spherical trigonometry to solve two spherical triangles. As previously indicated, Achintya Pal proposed a second approach that use three-dimensional algebraic coordinate geometry to arrive to the same equations. Coordinates are also known as axes Oxyz. O is the assumed unit-radius location of the Earth's center. The z-axis is OP. Although the Ox and Oy axes are not shown, it is possible to assume that the x-axis is pointing someplace behind the image (away from the viewer), and the y-axis is pointing somewhere in front of the painting. Both are, of course, in the plane of the equator.

Let us write the equation to the plane containing A and B in the form,

$$lx + my + nz = 0$$

Here (l, m, n) are the direction cosines of the normal to the plane AB, and are given by,

$$l = \sin\theta_0 \cos\phi_0 \qquad m = \sin\theta_0 \sin\phi_0 \qquad n = \cos\theta_0$$

The (x, y, z) coordinates of the point A are,

$$x = \sin\theta_1 \cos\phi_1 \qquad y = \sin\theta_1 \sin\phi_1 \qquad x = \cos\theta_1$$

On substitution of equations 3.5.21a, b,c and 3.5.22a,b,c into equation 3.5.20 we obtain:

$$\sin\theta_0 \cos\phi_0 \sin\theta_1 \cos\phi_1 + \sin\theta_0 \sin\phi_0 \sin\theta_1 \sin\phi_1 + \cos\theta_0 \cos\theta_1 = 0 \qquad 3.5.23$$

We quickly reach equation 3.5.17 again with some very little algebraic adjustments (for example, start by dividing by 1 0 sin 0 cos), and equation 3.5.18 follows suit.

As an added benefit, we see that the equation is satisfied for any location with spherical coordinates) (, situated on the great circle entire pole is at.

$$\cot\theta = -\tan\theta_0 \cos(\phi - \phi)$$

This equation may be thought of as the equation to the great circle AB since it provides the opposite solution to the original issue [9], [10].

CONCLUSION

A discussion of azimuth's mathematical underpinnings and how it relates to horizontal coordinates like height and zenith distance. For celestial navigation and stargazing, it is crucial to comprehend azimuth angle coordinates, especially in relation to other celestial coordinates. Additionally, we focused on real-world applications, emphasizing the crucial role azimuth plays in industries including surveying, mapping, and terrestrial navigation. Land surveyors and navigators use it as a compass to assist them identify direction and draw precise maps. Azimuth coordinates are essential for celestial observers in the field of astronomy because they allow them to identify and follow celestial objects as they travel across the sky. Astronomers may use the azimuth angle as a celestial compass to direct them to their intended objects and ensure correct observations. As we come to a conclusion in our exploration of azimuth angle coordinates, we encourage readers to recognize the usefulness and importance of this angular measuring method. The azimuth angle coordinates are an essential tool for comprehending and orienting oneself in both the terrestrial and celestial domains, whether you're a mariner utilizing them to navigate open seas, a surveyor charting the terrain, or an astronomer studying the cosmos. It serves as a reminder that, wherever they are used, angles and direction are crucial to our comprehension of the universe.

REFERENCES:

[1] A. Du and Y. Wang, "Compensation for Azimuth Angle or Scale Effect on Building Extraction in Urban Using SAR Scales of Polarization and Coherence," *IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens.*, 2016, doi: 10.1109/JSTARS.2016.2537269.

[2] W. G. Le Roux, "Optimum tilt and azimuth angles for fixed solar collectors in South Africa using measured data," *Renew. Energy*, 2016, doi: 10.1016/j.renene.2016.05.003.

[3] P. Blanc and L. Wald, "On the effective solar zenith and azimuth angles to use with measurements of hourly irradiation," *Adv. Sci. Res.*, 2016, doi: 10.5194/asr-13-1-2016.

[4] H. Singh, C. Sirisamphanwong, and S. M. Santhi Rekha, "Effect of Tilt and Azimuth Angle on the Performance of PV Rooftop System," *Appl. Mech. Mater.*, 2016, doi: 10.4028/www.scientific.net/amm.839.159.

[5] K. Sun, L. Zhang, and H. He, "Numerical computation on self-starting position of a three-blade vertical axis hydro-turbine," *Harbin Gongye Daxue Xuebao/Journal Harbin Inst. Technol.*, 2016, doi: 10.11918/j.issn.0367-6234.2016.04.024.

[6] L. Yang, S. Liu, D. Li, Q. Jian, and H. Cao, "Fast 2D DOA estimation algorithm by an array manifold matching method with parallel linear arrays," *Sensors (Switzerland)*, 2016, doi: 10.3390/s16030274.

[7] K. kai Xu, Z. yin Xiao, J. yao Tang, D. jun Liu, X. long Ma, and Z. hua Wang, "Dispersionless and Giant Optical Activity in Terahertz Chiral Metamaterials," *Plasmonics*, 2016, doi: 10.1007/s11468-015-0169-y.

[8] Y. Suzuki, S. Araki, Y. Baba, T. Tsuboi, S. Okabe, and V. A. Rakov, "An FDTD study of errors in magnetic direction finding of lightning due to the presence of conducting structure near the field measuring station," *Atmosphere (Basel).*, 2016, doi: 10.3390/atmos7070092.

[9] S. Wu *et al.*, "Study on a Novel UWB Linear Array Human Respiration Model and Detection Method," *IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens.*, 2016, doi: 10.1109/JSTARS.2016.2519760.

[10] R. D. Lindsley and D. G. Long, "ASCAT and QuikSCAT Azimuth Modulation of Backscatter over East Antarctica," *IEEE Geosci. Remote Sens. Lett.*, 2016, doi: 10.1109/LGRS.2016.2572101.

CHAPTER 9

EXPLORING THE INTRICATE WORLD OF METEOR ASTRONOMY

Puneet Sethi, Associate Professor
Department of Management, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- sethipuneet5@gmail.com

ABSTRACT:

Meteor Astronomy is a captivating field of study that focuses on the observation and analysis of meteors, the dazzling streaks of light that grace our night skies when tiny particles enter the Earth's atmosphere and burn up due to friction. This chapter explores the intricate world of meteor astronomy, shedding light on the origins, classifications, and dynamics of meteors. We delve into the science of meteor showers, examining the celestial phenomena that give rise to these spectacular displays. Moreover, we explore the crucial role of meteor astronomy in understanding the early solar system, tracing the origins of life's building blocks, and assessing the risks posed by near-Earth objects.

By the chapter's conclusion, readers will have gained a profound appreciation for the beauty and scientific significance of meteor astronomy and its contributions to our understanding of the universe we've ventured into the captivating realm of meteor astronomy, where the night sky becomes a canvas painted with streaks of light, offering insights into the origins of our solar system and the cosmic events that shape our world.

We began by exploring the origins of meteors, those brilliant flashes of light that grace our atmosphere when cosmic debris, often no larger than a grain of sand, hurtles through space and incinerates upon entry.

The science of meteor astronomy unravels the mysteries of these celestial phenomena, shedding light on their compositions, trajectories, and the dynamic processes that lead to their luminous displays.

KEYWORDS:

Astronomy, Celestial, Debris, Meteor, Meteor Astronomy, Science.

INTRODUCTION

The formula 1 2 1 2 1 2 cos = x x + y y + z z yields the angle between r1 and r2. Here, this outcome is not necessary. It is merely included here to demonstrate that the first example, which involves determining the distance between observatories in Victoria and Toronto, may also be solved using the same approach [1], [2].

The direction ratios of the line through the origin that is perpendicular to the plane containing the vectors r1 and r2 are (l, m), (l,1), and (l, m). Next, we have:

$$lx_1 + my_1 = z_1$$
$$lx_2 + my_2 = z_2.$$

Thus 1 1 1 lx + my = z is the equation to the plane passing through the origin and containing r1 and r2; i.e., the great circle passing through A and B lies at the intersection of the unit sphere with this plane. In our particular numerical example, the solution of the above two equations gives l = −3.21852823 and m = +7.763383.

The unit vector **p** this is one of the poles being sought is obtained by normalizing the direction ratios by $\sqrt{l^2 + m^2 + 1}$. Let (θ_0, ϕ_0) denote the colatitude and longitude of **p**. Then

$$p = \left(\frac{l}{\sqrt{l^2 + m^2 + 1}}, \frac{m}{\sqrt{l^2 + m^2 + 1}}, \frac{1}{\sqrt{l^2 + m^2 + 1}} \right)$$

$$= (\sin\theta_0 \cos\phi_0 \ , \sin\theta_0 \sin\phi_0, \cos\theta_0).$$

From the equality of the third component, we obtain $83^{\circ}\ 12'.9$ (i.e. latitude $6^{\circ}\ 47'.1$ N). From the first and second components, we have $\tan\phi_0 = \frac{m}{l}$ and hence $\phi_0 = 112^{\circ}\ 31'.1$ E. The other pole is at latitude $6^{\circ}\ 47'.1$ S, longitude $292^{\circ}\ 31'.1$ E

Here is a difficult but crucial practice in meteor astronomy. There are two meteors from the shower that diverge from a shared radiant. One begins at right ascension six hours and declination plus sixty-five degrees and ends at right ascension one hour and declination plus seventy-five degrees. The second begins at 5 hours' right ascension and 35 degrees' declination and ends at 3 hours' right ascension and 15 degrees' declination. The dazzling whereabouts? The diligent learner will precisely depict the scenario in the celestial realm via excellent sketching. Spherical triangles will need to be creatively manipulated for the computation. Look at you're drawing to see if it makes sense once you've come up with what you perceive to be the right solution. The next step can include writing a generic trigonometrical equation for the result in terms of the input data or programming the calculation into a computer so that it can be used going forward for any calculations of a similar kind. Another option is to create a computer program that will provide a least-squares solution for the radiant for the shower's many more meteors than just two. I discover that the radiant is at right ascension 7.26 hours and declination +43.8 degrees to provide the solution to the aforementioned puzzle [3], [4].

Uniqueness of Solutions

The reader who has already solved a number of triangle-related issues would have seen that, given three triangle's elements, sometimes there is only one solution and other times there are two potential triangles that meet the initial data. Once again, it is sometimes discovered that there is no feasible solution, which means that there is no triangle that could possibly satisfy the provided data and is thus assumed to be wrong. I owe Alan Johnstone a great deal for our extended talks of this issue and for pointing out that some of the "solutions" provided in an earlier draft of these notes were really false (and have now been updated). For plane triangles and spherical triangles, I think the following factors affect how many valid solutions there are for a given triplet of data [5], [6].

We may be given three elements of a triangle,

Thus

i. Three sides: a, b, c,

ii. Two sides and the included angle: b, c, A.

iii. Two sides and a nonincluded angle: a, b, A.

iv. Two angles and a common side: a, B, C.

v. Two angles and another side: A, B, a.

vi. Three angles: A, B, C.

Question:

Which of these give a unique solution, and which admit of two solutions? And which are impossible triangles? I believe the answers are as follows:

Plane Triangles

i. Let $d = a + b - c$, $e = b + c - a$, $f = c + a - b$

For a valid triangle, *d*, *e*, and *f* must all be positive. If so, there is a unique solution.

ii. There is a unique solution.

iii. If $a > b$ there is a unique solution.

If $a = b$, there is a unique solution if $A < 90°$. Otherwise there is no valid triangle.

If $a < b$ there are zero, one or two solutions, according as to whether $\sin A > \frac{a}{b}$, $\sin A = \frac{a}{b}$ or $\sin A < \frac{a}{b}$.

iv. There is a unique solution.

1. There is a unique solution.
2. There is a unique solution except that only the relative lengths of the sides are determined.

Rotation of Axes, Two Dimensions

We examine the following issue in this part. Take into account two sets of orthogonal axes, Ox, Oy and Ox', Oy, where Ox, Oy creates an angle with Oy, Oy'. Below is Figure 1. Either the coordinates of a point P with regard to one "basis set" Ox, Oy, or the coordinates of that point with respect to the other basis set Ox', Oy', may be used to describe that point. What is the relationship between (x, y) coordinates and (x', y') coordinates?

We see that OA = x, AP = y, ON = x', PN = y', OM = $x\cos\theta$, MN = $y\sin\theta$,

$$\therefore \quad x' = x\cos\theta + y\sin\theta. \qquad 3.6.1$$

Also MA = NB = $x\sin\theta$, PB = $y\cos\theta$,

$$\therefore \quad y' = -x\sin\theta + y\cos\theta. \qquad 3.6.2$$

These two relations can be written in matrix form as

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \qquad 3.6.3$$

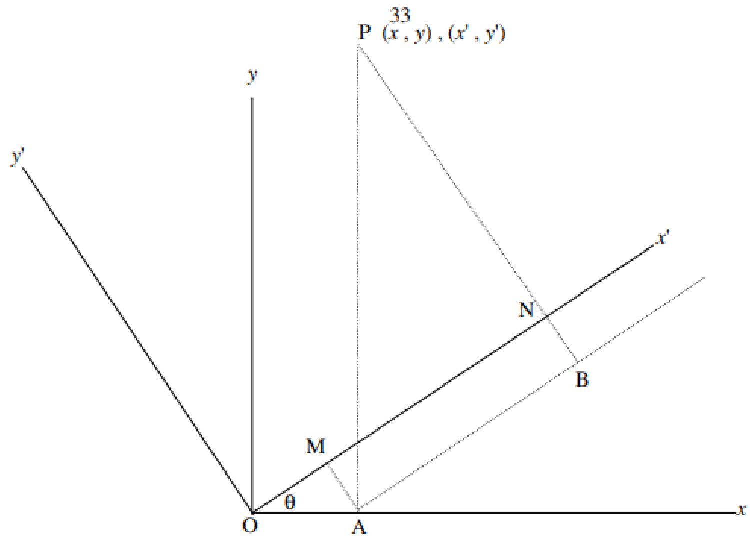


Figure 1: Illustrates theRelation Between the Coordinates x-axis and y-axis.

There are various methods for constructing equations for x and y in terms of x' and y', or the converse relations. The reader is urged to create drawings that are comparable to (b) and (c) and clearly depict the converse connections as one method. Another approach is to just solve the two equations above for x and y, which may be thought of as two simultaneous equations in x and y. It is less time-consuming to switch the primed and unprimed symbols and alter the sign of. The easiest of all is probably to understand that the matrix's determinant [7], [8].

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

is one, indicating that the matrix is an orthogonal one. It's a crucial characteristic of an orthogonal matrix M that its reciprocal M 1 equals its transpose M, which is created by flipping the rows and columns. As a result, the opposite relationship that we desire is,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}. \qquad 3.6.4$$

The reader may want to test each of the four approaches to be sure they all get the same outcome.

Rotation of Axes, Three Dimensions. Eulerian Angles

Now let's look at two sets of orthogonal axes that are inclined to one another in three dimensions: Ox, Oy, Oz and Ox', Oy, Oz. In terms of one basis set, a point in space may be characterized by its coordinates (x, y, and z) or (x', y, and z') in terms of the other. What connection exists between the coordinates (x, y, z) and (x', y, z')?

We must first specify the precise angle at which the primed and unprimed axes are slanted. The axes Ox, Oy, and Oz are shown in the following image. The axes Ox' and Oz' are also shown, but the axis Oy' is not represented since it is pointed behind the plane of the paper. The three angles, and, also referred to as the Eulerian angles, are shown in Figure 2 and indicate how the primed axes are oriented in relation to the unprimed axes. By doing three successive rotations, it is possible to comprehend the three angles' exact meanings [9], [10].

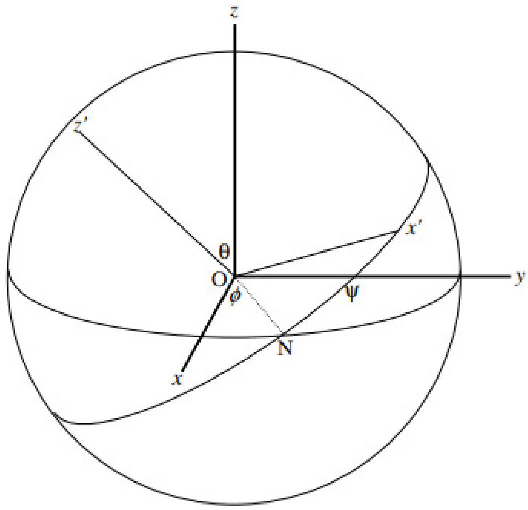


Figure 2: Illustrates the three angles can be understood by three consecutive rotations.

As illustrated in Figure 3, the Oz axis is first rotated through counterclockwise to create a series of intermediate axes called Ox1, Oy1, and Oz1. The axes of Oz and Oz1 are the same. The rotation is shown in Part (b) while gazing straight down the Oz (or Oz1) axis.

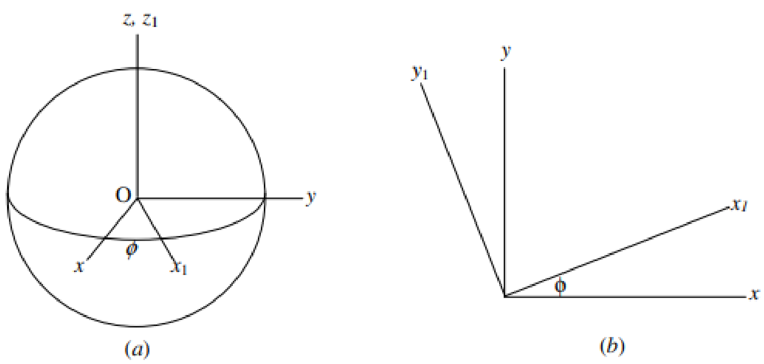


Figure 3: Illustrates the rotation of circle in three-dimension angle.

The relation between the (x, y, z) and (x1, y1, z1) coordinates is,

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \qquad 3.7.1$$

The Ox1 axis is then rotated counterclockwise to create the Ox2, Oy2, and Oz2 axes. Ox1 and Ox2 axes are interchangeable. The rotation is shown in part (b) of the diagram when gazing straight at the origin along the Ox1 (or Ox2) axis.

The relationship between the coordinates (x1, y1, z1) and (x2, y2, z2) is,

$$\begin{pmatrix} y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} y_1 \\ z_1 \end{pmatrix}.$$

Lastly, a rotation through ψ counterclockwise around the Oz2 axis to form the set of axes Ox', Oy' Oz'. The Oz2 and Oz' axes are identical. Part (b) of the figure shows the rotation as seen when looking directly down the Oz2 (or Oz') axis,

The relation between the (x1, y1, z1) and (x2, y2, z2) coordinates is,

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 1 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}.$$

Thus we have for the relations between (x',y',z') and (x,y,z).

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \qquad 3.7.4$$

On multiplication of these matrices, we obtain

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \qquad 3.7.5$$

The inverse of this can be discovered, as in the case of two dimensions, by solving these three equations for x, y, and z (which would be rather tedious); by exchanging primed and unprimed quantities; by flipping the order and signs of all operations; or by realizing that the determinant of the matrix is unity and that its reciprocal is its transpose; which is hardly tedious at all. By multiplying it out and using trigonometric identities, the reader should confirm that the matrix's determinant is one. However, because the magnitude of a vector cannot be altered by axis rotating, the determinant must be unity and the rotation matrix must be orthogonal. The cosine of the angle between an axis in one basis set and an axis in the other basis set makes up each member of the matrix. For instance, the cosine of the angles between Ox' and Oy is the second element in the first row. The cosine of the angles between Oz' and Ox is the first component in the third row. The relationships between the coordinates may be expressed by writing the matrix as a matrix of direction cosines between the axes of one basis set and the axes of the other basis set,

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad 3.7.6$$

$$\mathbf{R'} = \mathbf{CR}. \qquad 3.7.7$$

You'll see that the direction cosine forms resemble the cosine formula used to solve spherical triangles, and all of the direction cosines may be obtained by drawing and solving the appropriate spherical triangles. You may (or might not!) find this enjoyable to try. The direction cosines matrix C is orthogonal, and an orthogonal matrix has the following characteristics. The formulae for the direction cosines in terms of the Eulerian angles should be used by the reader to confirm this.

CONCLUSION

We explored the fascinating realm of meteor showers, regular celestial occurrences when Earth collides with cometary debris trails, resulting in a plethora of meteors flashing across the night sky.

These showers provide an amazing display that astronomers and stargazers alike flock to see and document. We also looked at meteor astronomy's enormous scientific relevance. Important hints concerning the creation and material distribution of our solar system may be found in meteors.

They are signs of precious resources and possible dangers, leading us to think about planetary defense tactics against impact occurrences. We ask readers to look up at the night sky with newfound awe and admiration as we come to the end of our trip through meteor astronomy. Meteors' brief flashes of light serve as a reminder of the continuing cosmic dance inside and beyond of our solar system.

With its fusion of celestial beauty and scientific curiosity, meteor astronomy continues to excite and enlighten our knowledge of the cosmos, providing a cosmic link that extends beyond the limits of our world.

REFERENCES:

[1] S. Vítek, P. Páta, P. Koten, and K. Fliegel, “Long-term continuous double station observation of faint meteor showers,” *Sensors (Switzerland)*, 2016, doi: 10.3390/s16091493.

[2] S. V. Kolomiyets, “Meteor Databases in Astronomy,” in *Proceedings of the International Astronomical Union*, 2016. doi: 10.1017/S1743921317000527.

[3] M. J. Martínez Usó and F. J. Marco Castillo, “A survey of meteoric activity over Spain during the eighth-fifteenth centuries,” *Journal for the History of Astronomy*. 2016. doi: 10.1177/0021828616649316.

[4] P. Pecina, “An analytical theory of a scattering of radio waves on meteoric ionization - II. Solution of the integro-differential equation in case of backscatter,” *Mon. Not. R. Astron. Soc.*, 2016, doi: 10.1093/mnras/stw1960.

[5] H. Kragh, “The source of solar energy, ca. 1840–1910: From meteoric hypothesis to radioactive speculations,” *Eur. Phys. J. H*, 2016, doi: 10.1140/epjh/e2016-70045-7.

[6] P. Borin, G. Cremonese, and F. Marzari, “Erratum: Statistical analysis of the flux of micrometeoroids at Mercury from both cometary and asteroidal components (Astronomy and Astrophysics (2016) A106 DOI: 10.1051/0004-6361/201526767),” *Astronomy and Astrophysics*. 2016. doi: 10.1051/0004-6361/201526767e.

[7] S. J. Schechner and D. Sliski, “The scientific and historical value of annotations on astronomical photographic plates,” *J. Hist. Astron.*, 2016, doi: 10.1177/0021828615624094.

[8] K. Barbora, “Questio astronomicalis zdeňka z labouně kritická edice,” *Listy Filologicke*. 2016.

[9] S. A. G. *et al.*, "Balloon UV experiments for astronomical and atmospheric observations," in *Ground-based and Airborne Instrumentation for Astronomy VI*, 2016. doi: 10.1117/12.2232246.

[10] K. Shalabi, S. Graziado, R. Davidson, G. Morgan, and J. Eyre, "Learning by chronic stroke survivors of 2 sequentially linked actions is preserved in dominant hemisphere strokes but may be impaired in non-dominant hemisphere strokes," *Int. J. stroke*, 2016.

CHAPTER 10

EULERIAN ANGLE: FUNDAMENTAL CONCEPT OF MATHEMATICS

Sunil Kumar Srivastava, Associate Professor
Department of Pharmacy, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- shyam.sunil32@gmail.com

ABSTRACT:

Eulerian angles, a fundamental concept in mathematics and engineering, provide a powerful framework for describing the orientation of objects in three-dimensional space. In this chapter, we embark on a journey through the intricate world of Eulerian angles, exploring their origins, properties, and wide-ranging applications.

We begin by introducing the basic concepts of Euler angles, which enable us to represent any arbitrary rotation in space. We then delve into the intricacies of different Euler angle conventions and their unique characteristics.

Throughout our exploration, we highlight the importance of Eulerian angles in fields such as aerospace engineering, robotics, computer graphics, and physics. By the chapter's conclusion, readers will have gained a deep understanding of Eulerian angles and their pivotal role in representing and manipulating orientations in 3D space we have delved into the rich world of Eulerian angles, unearthing their significance and versatility in representing rotations and orientations in three-dimensional space.

We began our journey by introducing the core concepts of Euler angles, emphasizing their role as a mathematical tool for characterizing the orientation of objects.

Eulerian angles offer a unique and intuitive way to describe complex rotations in 3D, breaking them down into a sequence of simpler rotations about the coordinate axes.

KEYWORDS:

Celestial, Coordinate, Cylindrical, Equatorial, Geographical, Geographic.

INTRODUCTION

Of course, the conditions also hold for the rotation matrix in two dimensions, although less strictly [1], [2].

(a) det $\mathbf{C} = \pm 1$
(det $\mathbf{C} = -1$ implies that the two basis sets are of opposite chirality or "handedness"; that is, if one basis set is right-handed, the other is left-handed.)

(b) The squares of all the items in any row or column add up to one. Simply put, this only confirms that the magnitudes of unit orthogonal vectors are one [3], [4].

(c) The sum of any two rows' or any two columns' products of identical components is zero. This simply reflects the fact that any two-unit orthogonal vectors' scalar or dot product is zero.

(d) Each element has an equal cofactor. This is a reflection of the fact that any two unit orthogonal vectors in cyclic order have a vector or cross product that is identical to the third.

(e) $\mathbf{C}^{-1} = \tilde{\mathbf{C}}$, or the reciprocal of an orthogonal matrix is equal to its transpose.

In a numerical scenario, the first four qualities listed above may be (and should be) used to check if the matrix is in fact orthogonal as well as to find and fix errors [5], [6].

The following matrix, for instance, is meant to be orthogonal, but there are really two errors in it. Find the errors using the aforementioned characteristics (b) and (c).

When you do this, it will become evident why verifying property (b) alone is insufficient. Check to check whether you can locate the Eulerian angles, and without quadrant ambiguity after you have adjusted the matrix. Start at the bottom right corner of the matrix and take notice of how the Eulerian angles are set up.

This will help you to see that must be between 0o and 180o, so there is no ambiguity of quadrant.

However, the other two angles must be identified by evaluating the signs of their sines and cosines, which may range from 00 to 3600.

A additional helpful activity would be to create a graphic depicting the orientation of the primed axes with respect to the unprimed axes after you have determined the Eulerian angles [7], [8].

$$\begin{pmatrix} +0.075\ 284\ 882\ 7 & -0.518\ 674\ 468\ 2 & +0.851\ 650\ 7396 \\ -0.553\ 110\ 473\ 2 & -0.732\ 363\ 000\ 8 & +0.397\ 131\ 261\ 9 \\ -0.829\ 699\ 337\ 5 & +0.442\ 158\ 963\ 2 & -0.342\ 020\ 143\ 3 \end{pmatrix}$$

As a matter of good computing practice, take note that all numbers, positive and negative, are signed, and leading zeroes are not removed.

The numbers are also written in groups of three, separated by half spaces after the decimal point. As a matter of good computing practice, take note that all numbers, positive and negative, are signed, and leading zeroes are not removed.

The numbers are also written in groups of three, separated by half spaces after the decimal point.

Formulas for trigonometry

I've included a list of frequently used trigonometric formulae here just for reference.

Whether you choose to memorize them is a question of choice. It is probably true to say that, whether or not a deliberate attempt was made to remember them, anybody who routinely engages in issues in celestial mechanics or similar fields will be acquainted with most of them, at least from frequent usage.

Even if the reader needs to search to remember the specific formula, they should at the very least be known to exist [9], [10].

$$\frac{\sin A}{\cos A} = \tan A$$

DISCUSSION

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \cot^2 A = \csc^2 A$$

$$1 + \tan^2 A = \sec^2 A$$

$$\sec A \csc A = \tan A + \cot A$$

$$\sec^2 A \csc^2 A = \sec^2 A + \csc^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin \tfrac{1}{2} A = \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{A + \cos A} = \csc A - \cot A$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2} S \cos \tfrac{1}{2} D,$$

where $S = A + B$ and $D = A - B$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2} S \sin \tfrac{1}{2} D$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2} S \cos \tfrac{1}{2} D$$

$$\cos A - \cos B = -2 \sin \tfrac{1}{2} S \sin \tfrac{1}{2} D$$

$$\sin A \sin B = \tfrac{1}{2}(\cos D - \cos S)$$

$$\cos A \cos B = \tfrac{1}{2}(\cos D + \cos S)$$

$$\sin A \cos B = \tfrac{1}{2}(\sin S + \sin D)$$

$$\sin A = \frac{T}{\sqrt{1 + T^2}} = \frac{2t}{1 + t^2},$$

where $T = \tan A$ and $t = \tan \tfrac{1}{2} A$

$$\cos A = \frac{1}{\sqrt{1 + T^2}} = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = T = \frac{2t}{1 - t^2}$$

$$s = \sin A, \quad c = \cos A$$

$$\cos A = c \qquad \sin A = s$$
$$\cos 2A = 2c^2 - 1 \qquad \sin 2A = 2cs$$
$$\cos 3A = 4c^3 - 3c \qquad \sin 3A = 3s - 4s^3$$
$$\cos 4A = 8c^4 - 8c^2 + 1 \qquad \sin 4A = 4c(s - 2s^3)$$
$$\cos 5A = 16c^5 - 20c^3 + 5c \qquad \sin 5A = 5s - 20s^3 + 16s^5$$
$$\cos 6A = 32c^6 - 48c^4 + 18c^2 - 1 \qquad \sin 6A = 2c(3s - 16s^3 + 16s^5)$$
$$\cos 7A = 64c^7 - 112c^5 + 56c^3 - 7c \qquad \sin 7A = 7s - 56s^3 + 112s^5 - 64s^7$$
$$\cos 8A = 128c^8 - 256c^6 + 160c^4 - 32c^2 + 1 \qquad \sin 8A = 8c(s - 10s^3 + 24s^5 - 16s^7)$$

$$\tan A = T$$
$$\tan 2A = \frac{2T}{1 - T^2}$$
$$\tan 3A = \frac{T(3 - T^2)}{1 - 3T^2}$$
$$\tan 4A = \frac{4T(1 - T^2)}{1 - 6T^2}$$
$$\tan 5A = \frac{T(5 - 10T^2 + T^4)}{1 - 10T^2 + 5T^4}$$

$$\tan nA = \frac{T\left(\binom{n}{1} - \binom{n}{3}T^2 + \binom{n}{5}T^4 - \ldots\right)}{1 - \binom{n}{2}T^2 + \binom{n}{4}T^4 - \ldots}, \qquad \binom{n}{r} = \text{binomial coefficient}$$

$$\cos^2 A = \tfrac{1}{2}(\cos 2A + 1)$$
$$\cos^3 A = \tfrac{1}{4}(\cos 3A + 3\cos A)$$
$$\cos^4 A = \tfrac{1}{8}(\cos 4A + 4\cos 2A + 3)$$
$$\cos^5 A = \tfrac{1}{16}(\cos 5A + 5\cos 3A + 10\cos A)$$
$$\cos^6 A = \tfrac{1}{32}(\cos 6A + 6\cos 4A + 15\cos 2A + 10)$$
$$\cos^7 A = \tfrac{1}{64}(\cos 7A + 7\cos 5A + 21\cos 3A + 35\cos A)$$
$$\cos^8 A = \tfrac{1}{128}(\cos 8A + 8\cos 6A + 28\cos 4A + 56\cos 2A + 35)$$

$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$
$$\sin^3 A = \tfrac{1}{4}(3\sin A - \sin 3A)$$
$$\sin^4 A = \tfrac{1}{8}(\cos 4A - 4\cos 2A + 3)$$
$$\sin^5 A = \tfrac{1}{16}(\sin 5A - 5\sin 3A + 10\sin A)$$
$$\sin^6 A = \tfrac{1}{32}(10 - 15\cos 2A + 6\cos 4A - \cos 6A)$$
$$\sin^7 A = \tfrac{1}{64}(35\sin A - 21\sin 3A + 7\sin 5A - \sin 7A)$$
$$\sin^8 A = \tfrac{1}{128}(\cos 8A - 8\cos 6A + 28\cos 4A - 56\cos 2A + 35)$$

$$\sin A = A - \frac{A^3}{3!} + \frac{A^5}{5!} - \cdots$$

$$\cos A = 1 - \frac{A^2}{2!} + \frac{A^4}{4!} - \cdots$$

$$\int_0^{\pi/2} \sin^m \theta \cos^n \theta \, d\theta = \frac{(m-1)!!(n-1)!!X}{(m+n)!!},$$ where $X = \pi/2$ if m and n are both even, and

$X = 1$ otherwise.

$e^{ni\theta} = e^{in\theta}$ (de Moivre's theorem - the only one you need know. All others can be deduced from it.)

Plane triangles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a\cos B + b\cos A = c$$

$$s = \tfrac{1}{2}(a+b+c)$$

$$\sin\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\cos\tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$

$$\tan\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

Spherical triangles

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\cos a = \cos b\cos c + \sin b\sin c\cos A$$

$$\cos A = -\cos B\cos C + \sin B\sin C\cos a$$

$$\cos(\text{IS})\cos(\text{IA}) = \sin(\text{IS})\cot(\text{OS}) - \sin(\text{IA})\cot(\text{OA})$$

CONCLUSION

Each of the several Euler angle conventions has its own orderings and definitions. These norms, which are modified for various applications and areas, demonstrate how flexible Eulerian angles are in a variety of contexts. Eulerian angles serve as a universal language for conveying and processing orientation data, whether in aeronautical engineering, robotics, computer graphics, or physics.

As a result of our investigation, we were able to identify certain possible drawbacks and difficulties related to Eulerian angles, such as gimbal lock, which is a phenomenon that happens when one of the angles becomes singular and results in the loss of one degree of freedom in the representation of rotations. Although these difficulties really exist, engineers and mathematicians have discovered solutions to lessen their effects.

Eulerian angles are a fundamental component of spatial orientation representation, bridging the conceptual gap between mathematics and real-world applications. They are a vital tool for anybody working in industries where comprehending and controlling orientations in three-dimensional space is crucial because of how elegantly simple and practical they are. As a long-standing and essential part of our mathematical toolset, Eulerian angles continue to improve science, technology, and engineering.

REFERENCES:

[1] D. B. Widyantara, R. Sumiharto, and S. B. Wibowo, "Purwarupa Sistem Kendali Kestabilan Pesawat Tanpa Awak Sayap Tetap Menggunakan Robust PID," *IJEIS (Indonesian J. Electron. Instrum. Syst.*, 2016, doi: 10.22146/ijeis.15260.

[2] N. Yoshioka *et al.*, "Weight-bearing three-dimensional computed tomography analysis of the forefoot in patients with flatfoot deformity," *J. Orthop. Sci.*, 2016, doi: 10.1016/j.jos.2015.12.001.

[3] S. Anvari, H. Taghavifar, S. Khalilarya, S. Jafarmadar, and M. T. Shervani-Tabar, "Numerical simulation of diesel injector nozzle flow and in-cylinder spray evolution," *Appl. Math. Model.*, 2016, doi: 10.1016/j.apm.2016.05.017.

[4] K. Roy, B. Hawlader, S. Kenny, and I. Moore, "Finite element modeling of lateral pipeline-soil interactions in dense sand," *Can. Geotech. J.*, 2016, doi: 10.1139/cgj-2015-0171.

[5] M. Mamourian, K. Milani Shirvan, S. Mirzakhanlari, and A. B. Rahimi, "Vortex generators position effect on heat transfer and nanofluid homogeneity: A numerical investigation and sensitivity analysis," *Appl. Therm. Eng.*, 2016, doi: 10.1016/j.applthermaleng.2016.07.068.

[6] H. Nowruzi and P. Ghadimi, "Effect of water-in-heavy fuel oil emulsion on the non-reacting spray characteristics under different ambient conditions and injection pressures: A CFD study," *Sci. Iran.*, 2016, doi: 10.24200/sci.2016.3972.

[7] T. Chakraborty, "Analysis of hollow steel piles subjected to buried blast loading," *Comput. Geotech.*, 2016, doi: 10.1016/j.compgeo.2016.05.015.

[8] Z. Wu and Y. Cao, "Numerical simulation of airfoil aerodynamic performance under the coupling effects of heavy rain and ice accretion," *Adv. Mech. Eng.*, 2016, doi: 10.1177/1687814016667162.

[9] W. Peng and X. Cao, "Numerical simulation of solid particle erosion in pipe bends for liquid-solid flow," *Powder Technol.*, 2016, doi: 10.1016/j.powtec.2016.02.030.

[10] M. Son, K. Yu, K. Radhakrishnan, B. Shin, and J. Koo, "Verification on spray simulation of a pintle injector for liquid rocket engine," *J. Therm. Sci.*, 2016, doi: 10.1007/s11630-016-0838-y.

CHAPTER 11

COORDINATE SYSTEMS AND COORDINATE TRANSFORMATIONS

Abhinav Saxena, Associate Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- abhinav.polytechnic@tmu.ac.in

ABSTRACT:

Coordinate Systems and Coordinate Transformations form the cornerstone of spatial representation and analysis across various fields, from mathematics and physics to engineering and geography. This comprehensive chapter takes readers on a journey through the intricate world of coordinate systems, elucidating their significance, principles, and practical applications. We begin by introducing the fundamental concept of coordinate systems, explaining how they define points in space and enable measurements and calculations. As we delve deeper, we explore the richness of different coordinate systems, including Cartesian, polar, and spherical systems, each tailored to specific needs. Furthermore, we demystify coordinate transformations, which allow us to seamlessly translate data from one system to another, facilitating interdisciplinary collaboration and problem-solving.

By the chapter's conclusion, readers will have gained a profound appreciation for the versatility and essential role of coordinate systems and transformations in understanding and navigating our multidimensional world we've embarked on a journey through the realms of coordinate systems and coordinate transformations, uncovering their fundamental importance and wide-ranging applications. We began by establishing the foundational concept of coordinate systems, which serve as the scaffolding for spatial representation in diverse disciplines. These systems provide a means to define the positions of points in space and facilitate the measurement and analysis of various phenomena.

KEYWORDS:

Coordinate Transformations, Geodesy, Mapping, Measurement, Navigation, Orientation.

INTRODUCTION

Topology, a branch of mathematics, provides a highly broad description of space. There are many unusual areas that lack an analog in the real world. In fact, the spaces that may be described by a coordinate system are among the most complex ones in the hierarchy of spaces defined by topology. actual scientists are drawn to these places because they resemble the actual environment in which we live. Such spaces are useful because they have a coordinate system that makes it possible to explain things that occur there. The locations of interest need not just be areas in the actual world, however. The thermodynamics' temperature-pressure-density space, among many other spaces where the dimensions are physical variables, may be seen.

Phase space is among these places that is crucial for mechanics. The location and momentum coordinates for a group of particles are stored in this multidimensional space. Physical places may therefore take many different shapes. They do, however, all share a characteristic. Some coordinate system or frame of reference is used to describe them. Picture a group of stiff rods or vectors that are all joined at a single point. A frame of reference is the term for such a group of "rods." The reference frame is said to span the space if each point in the space can be uniquely projected onto the rods, resulting in a distinctive group of rod-points identifying the point in space [1], [2].

Systems of orthogonal coordinates

The coordinate frame is said to be orthogonal if the vectors defining it are locally perpendicular. Imagine a collection of unit basis vectors e i that are spread out across a certain area. The concept of orthogonality may be expressed by:

$$\hat{e}_i \bullet \hat{e}_j = \delta_{ij} \quad , \qquad (2.1.1)$$

When a set of basis vectors spans a space with n dimensions, where n is the number of vectors e i, the set of basis vectors is said to be orthogonal. Notably, the space does not have to be Euclidean. However, the coordinate frame is referred to be a Cartesian frame if the space is Euclidean and it is orthogonal. A Cartesian frame is the typical x, y, and z coordinate frame. Such a coordinate frame may be drawn on a rubber sheet. The coordinate frame may remain orthogonal even if the space may no longer be a Euclidean space if the sheet is deformed in such a way that the local orthogonality constraints are still satisfied. Take latitude and longitude, which are the standard coordinates for the earth's surface. Although the surface is not on the Euclidean plane and the coordinates are not Cartesian, these coordinates are in fact orthogonal [3], [4].

There are various orthogonal coordinate systems that are often used to describe the physical world. The Cartesian or rectangular coordinate system (xyz) is unquestionably the most popular. The spherical or polar coordinate system (r,) is perhaps the second most popular and crucial for astronomy. The cylindrical coordinates (r, z) are less frequent but nonetheless highly significant. Laplace's equation may be solved for potential theory issues by knowing that there are thirteen orthogonal coordinate systems in which it can be separated (see Morse and Feshbackl). Recently, using ellipsoidal coordinates and certain potentials established there, it has become possible to understand the dynamics of ellipsoidal galaxies in a semi-analytic way. Although the mathematical physicists of the nineteenth century were primarily interested in these more esoteric coordinates, they are still relevant today. The selection of the appropriate coordinate system in which to conduct the analysis is sometimes the most crucial step in addressing a problem in mathematical physics.

One needs do more than simply provide the space and coordinate geometry in order to fully define any coordinate system. The coordinate system's origin and orientation must also be specified. There are three key sites for the genesis in celestial mechanics. In terms of observation, the observer may be regarded as the origin (topocentric coordinates). However, in order to interpret the observations, it is typically necessary to refer to coordinate systems that have their origins at the centers of the earth, the sun, or the solar system's mass (barycentric coordinates, heliocentric coordinates, and geocentric coordinates, respectively). Only when comparing or transforming the coordinate frame to another coordinate frame does the orientation become significant. This is often accomplished by defining the relative orientation as well as the zero-point of a coordinate with regard to the coordinates of the other frame [5], [6].

Systems of astronomical coordinates

Nearly all of the important coordinate systems in astronomy are spherical coordinate systems. Most astronomical objects are far from the earth, which causes them to seem to move against the background of the celestial sphere. This is the obvious cause of the problem. For adjacent objects, it is still possible to utilize a spherical coordinate system, but in order to prevent parallax issues, it may be required to designate the origin as the observer. Only the origin's position and the frames' relative orientation to one another will be different between these orthogonal coordinate frames. They are connected to the direction of the earth's rotation axis with regard to the stars and sun since they are based on observations obtained from the earth. The Right Ascension -Declination coordinate system is the most significant of these coordinate systems.

The Coordinate System for Right Ascension and Declination

This coordinate system is a spherical-polar coordinate system where the polar angle is measured from the equatorial plane of the system rather than the coordinate system's axis. As a result, the declination is the polar angle's angular counterpart. Simply described, it is the angular separation between the astronomical object and the celestial sphere measured north or south from the earth's equator. The origin of the coordinate system may be assumed to be the center of the earth for measurements of far-off objects collected from the earth. At least the coordinate system's 'azimuthal' angle is calculated correctly. In other words, if one points the fingers of his right hand in the direction of rising Right Ascension, the thumb of his right hand will point toward the North Pole. Some people recall it by seeing how rising or ascending stars' Right Ascensions go higher over time. Some people have a propensity to gaze south and believe the angle should rise to their right, as if they were gazing at a map. The idea so perplexed air force navigators during the Second World War that the complimentary angle, also known as the sidereal hour angle, was developed since this is precisely the opposite of the actual situation. The Right Ascension is just 24 hours away from this angular location.

The fact that this Right Ascension is not expressed in a conventional angular unit like degrees or radians is another facet of it that many people find puzzling. Instead, it is expressed in hours, minutes, and seconds. These units, however, are the natural ones since any stationary point in the sky will return to its original location after about 24 hours due to the earth's revolution on its axis. The zero-point from which the right ascension angle is measured must still be established. The direction of the planet is another inspiration for this. The sun's annual path serves as a description of how the earth's orbital plane is projected onto the celestial sphere. The ecliptic is the name of this journey. The ecliptic and equator, which are shown as large circles on the celestial sphere, intersect at two sites 180 degrees apart because the earth's rotation axis is inclined to the orbital plane. The locations are referred to as equinoxes because, when the sun is at them, it will be in the plane of the earth's equator and there will be an equal amount of day and darkness. Once a year, the sun will make a trip to each, once while it is traveling north along the ecliptic and once when it is traveling south. The first is known as the vernal equinox because it ushers in spring in the northern hemisphere, while the second is known as the autumnal equinox. The right ascension of an astronomical object is determined by measuring its eastward distance in hours, minutes, and seconds from the vernal equinox, which is the zero-point of the right ascension coordinate [7], [8].

While the center of the earth may be assumed to represent the origin of the coordinate system, the center of the sun can also be assumed. In this case, the coordinate system may be thought of as being simply adjusted until its origin coincides with the center of the sun without affecting its orientation. The study of stellar kinematics benefits from the usage of such a coordinate system. Some stellar dynamics investigations call for the use of a coordinate system whose origin is the earth-moon system's center of mass. Barycentric coordinates are what they are called. In fact, as "barycenter" refers to the center of mass, "barycentric coordinates" may also be used to describe a coordinate system whose origin is the solar system's center of mass. This origin will be extremely close to, but not the same as, the origin of the heliocentric coordinate system due to the sun's dominance over the solar system. Even while the variations in origin between heliocentric and barycentric coordinates are small, they are substantial enough to affect certain issues, such the timing of pulsars.

DISCUSSION

Since the motion of planets and asteroids, with a few noteworthy exceptions, is restricted to the zodiac, the ecliptic coordinate system is mostly utilized in research involving these objects. It has a lot of similarities conceptually with the right ascension-declination coordinate system. Instead of the equator, the ecliptic serves as the defining plane, and the "azimuthal" coordinate, which is often defined in degrees, is measured in the same direction. Although these titles would be more fitting for Declination and Right Ascension, the polar and azimuthal angles are given the rather unpleasant names of celestial latitude and celestial longitude, respectively. Once again, these coordinates may be topocentric, geocentric, heliocentric, or barycentric in nature.

System of Altimeter Coordinates

The coordinate system that most people are acquainted with is altitude-azimuth. This coordinate system's origin is the observer, and it is not often moved to another place. The observer and the horizon are both present in the system's basic plane. Although the idea of the horizon seems intuitively evident, a precise definition is required since the visible horizon seldom coincides with the real horizon's position. It must first be defined in order to define the zenith. It is more precisely described as the extension of the local gravity vector across the celestial sphere outward from the point immediately above the observer's head. The astronomical zenith is this location. This zenith is often near to the extension of the local radius vector from the center of the earth via the observer to the celestial sphere, with the exception of the earth's oblateness. The local gravity vector may deviate even more from the local radius vector in the presence of adjacent massive masses (such as a mountain). The line on the celestial sphere that is always 90 degrees away from the zenith is known as the horizon. An object's height is the angle at which it is above or below the horizon as measured along a large circle that passes between it and the zenith. The object's azimuth is the only angle that matters in this coordinate system for azimuth. The zero point's position is the sole issue in this situation. The azimuth is measured westward from the southmost point of the horizon, according to several older astronomy publications. However, only astronomers did this, and the majority no longer do. Everybody who uses local coordinate systems, including surveyors, pilots, and navigators, measures azimuth from the north horizon point, increasing via the east horizon point, and finally turning around to the west. I adopt that viewpoint throughout this work. As a result, the azimuth of the compass's cardinal points is as follows: N (0°), E (90°), S (18°), and W (270°).

Spatial Reference Systems

Before we move on from the discussion of specialized coordinate systems, we need talk about the coordinate systems used to measure the earth's surface. The form of the earth is most closely comparable to that of an oblate spheroid. The meaning of "local vertical" may be affected by this [9], [10].

System of Astronomical Coordinates

Latitude-longitude coordinates are the standard method for identifying locations on the surface of the world. The latitude is simply the angular distance north or south of the equator measured along the local meridian toward the pole, and the longitude is the angular distance measured along the equator to the local meridian from some reference meridian. This system is one that most people are familiar with. This reference meridian has traditionally been regarded as being determined by a particular device (the Airy transit) situated at Greenwich, England. The International Astronomical Union has lately established a convention according to which longitudes measured east of Greenwich are regarded as positive and those measured to the west as negative. Such coordinates provide a correct comprehension of an earth that is fully spherical. But extra caution must be used since the earth is not perfectly spherical.

Geographical Coordinate System

A coordinate system that approximates the form of the globe by an oblate spheroid has been developed in an effort to account for a non-spherical world. An ellipse may be rotated around its minor axis, which becomes the coordinate system's axis, to produce such a figure. The ellipse's equator is therefore the plane that the primary axis of the ellipse sweeps out. This model of the earth's true form is really fairly accurate. The angle between the local vertical and the equator, where the local vertical is the normal to the oblate spheroid at the location in question, now determines the geodetic latitude. The geodetic longitude is the angle between the local meridian and the meridian at Greenwich, and it is nearly equivalent to the longitude in the astronomical coordinate system. The "deflection of the vertical" is the difference between the local vertical, or the normal to the local surface, and the astronomical vertical, which is determined by the local gravity vector. This difference is often less than 20 arc seconds. A third coordinate system, also referred to as the geocentric coordinate system, might be introduced due to the flatness of the earth.

Geographical Coordinate System

Think about the oblate spheroid that most closely resembles the shape of the earth. Consider now a radius vector from the spheroid's center to any point on its surface. Except at the poles and the equator, the radius vector will typically not be perpendicular to the surface of the oblate spheroid, defining a distinct local vertical. The definition of a separate latitude from the astronomical or geodetic latitude may then be made using this. The highest discrepancy between geocentric and geodetic latitudes for the planet is (11' 33"), and it occurs at a latitude of roughly 45°. This may not seem like much, but on the surface of the globe, it equates to around eleven and a half nautical miles (13.3 miles or 21.4 kilometers). So, you must be attentive while choosing a coordinate system if you actually want to know where you are. Similar to the geodetic longitude, the geocentric longitude is defined as the angle between the local meridian and the meridian at Greenwich.

Transformative Coordination

The practical component of celestial mechanics requires converting observable values between different coordinate systems in a significant way. In order to identify the laws that relate to the issues we will experience in celestial mechanics; it is important that we examine how this is done generally. Although there are many coordinate transformations that may be defined within the context of mathematics, we will focus on a distinct subset known as linear transformations. These coordinate transformations use a sequence of linear algebraic equations to connect the coordinates in one frame to those in another. Thus if a vector X in one coordinate system has components Xj, in a primed-coordinate system a vector X' have components Xj given by:

$$X_i' = \sum_j A_{ij} X_j + B_i$$

In vector notation we could write this as,

$$\vec{X}' = \mathbf{A}\vec{X} + \vec{B} \quad .$$

With A being a matrix and B being a vector, this establishes the general class of linear transformations. The matrix A and the vector B are the two components that make up this generic linear form. It is evident that the vector B may be seen as a change in the coordinate system's origin, but the components Aij in Figure 1 are the directions cosines, or cosines of the angles between the axes Xi and Xj. A vector from the origin of the unprimed coordinate frame to the origin of the primed coordinate frame is all that the vector B really is. Now, if we take two fixed locations in space and a vector linking them, the direction and length of that vector will not rely on the origin of the coordinate system used to make the measurements. The kinds of linear transformations we can take into consideration are further limited by this. For instance, although linear, scaling each coordinate by a certain amount would alter the vector's length as determined by the two coordinate systems. The length of the vector must be independent of the coordinate system as we are simply using it to conveniently describe the vector. Therefore, we will limit the linear transformations that we study to those that change orthogonal coordinate systems while maintaining the length of the vector.

Thus the matrix **A** must satisfy the following condition

$$\vec{X}' \bullet \vec{X}' = (\mathbf{A}\vec{X}) \bullet (\mathbf{A}\vec{X}) = \vec{X} \bullet \vec{X} \quad , \qquad (2.4.3)$$

which in component form becomes

$$\sum_i \Big(\sum_j A_{ij} X_j\Big)\Big(\sum_k A_{ik} X_k\Big) = \sum_j \sum_k \Big(\sum_i A_{ij} A_{ik}\Big) X_j X_k = \sum_i X_i^2 \quad . \qquad (2.4.4)$$

This must be true for all vectors in the coordinate system so that

$$\sum_i A_{ij} A_{ik} = \delta_{jk} = \sum_i A_{ji}^{-1} A_{ik} \quad . \qquad (2.4.5)$$

Remember that the Kronecker delta ij is the unit matrix and that the inverse of any element inside a group that multiplies another to create the group's unit element is defined by the Kronecker delta ij. Therefor,

$$A_{ji} = [A_{ij}]^{-1}$$

A new matrix is created by switching around a matrix's components; this new matrix is known as the transpose of the matrix. As a result, inverses of orthogonal transformations that maintain the length of vectors are just the original matrix transposed,

Thus, given the transformation A in the linear system of equations (2.4.2), we may either reverse the transformation or solve the linear equations by multiplying those equations by the transposed original matrix,

$$\vec{X} = \mathbf{A}^{\mathbf{T}} \vec{X}' - \mathbf{A}^{\mathbf{T}} \vec{B}$$

These are known as orthogonal unitary transformations or orthonormal transformations, and the conclusion provided in equation (2.4.8) substantially simplifies the process of transforming from one coordinate system to another and back again. We may further categorize orthonormal transformations into two types. These are best conveyed by picturing the relative orientation of the two coordinate systems.

Consider a transformation that changes one coordinate in the new coordinate system to the inverse of its counterpart while keeping the rest unaltered. If the modified coordinate is, for example, the x-coordinate, the transformation matrix,

$$\mathbf{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}^{-1} = \mathbf{A}^{\mathrm{T}} \quad .$$

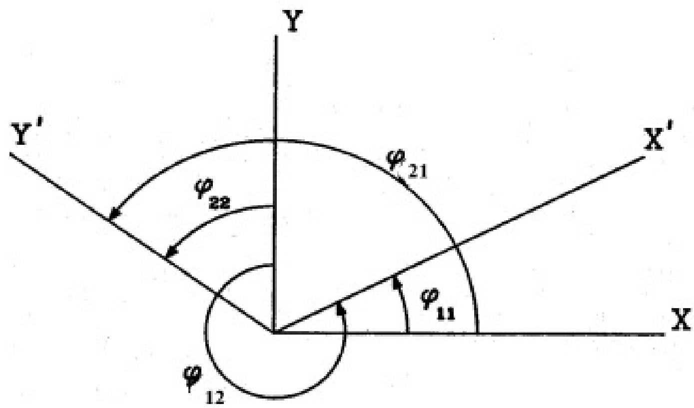


Figure 1: shows how two coordinate frames related by the transformation angles. Four coordinates are necessary if the frames are not orthogonal.

This is analogous to looking at the first coordinate system via a mirror. These transformations, known as reflection transformations, convert a right-handed coordinate system to a left-handed coordinate system. Any vectors' lengths will stay constant. In the new coordinate system, the x-component of these vectors will simply be substituted by its negative. This will not be the case for "vectors" produced by the vector cross product. The values of such a vector's components will stay intact, suggesting that a reflection transformation of such a vector would alter its orientation. This is the "right hand rule" for vector cross products, if you will. A vector pointing in the opposite direction is produced by a left-hand rule. As a result, such vectors are not invariant to reflection transformations since their orientation changes, which is why they are classified as axial (pseudo) vectors. The Levi-Civita tensor must have this unusual transformation feature because it creates the vector cross product from the constituents of ordinary (polar) vectors. Tensors with this transformation feature are often referred to as tensor densities or pseudo-tensors. As a result, we should refer to the Levi-Civita tensor density as stated in equation (1.2.7).

Indeed, the invariance of tensors, vectors, and scalars to orthonormal transformations is the most accurate way to define the components of the tensor group. Finally, it is worth noting that the determinant of an orthonormal reflection transformation is -1. The unitary magnitude of the determinant results from the vector's magnitude being unaffected by the transformation, however the sign indicates that some combination of coordinates has been reflected. The components of the second class of orthonormal transformations have determinants of +1, as one would anticipate. These are transformations that may be thought of as a rotation of the coordinate system around an axis. Consider the transition between the two coordinate systems.1. In the primed coordinate system, the components of any vector C are given by:

$$\begin{pmatrix} C_{x'} \\ C_{y'} \\ C_{z'} \end{pmatrix} = \begin{pmatrix} \cos\phi_{11} & \cos\phi_{12} & 0 \\ \cos\phi_{21} & \cos\phi_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix}$$

If the transformation must be orthonormal, the direction cosines will not be linearly independent since the angles between the axes must be /2 in both coordinate systems. As a result, the angles must be connected by:

$$\left.\begin{array}{l} \phi_{11} = \phi_{22} = \phi \\ \phi_{21} = \phi_{11} + \pi/2 = \phi + \pi/2 \\ (2\pi - \phi_{12}) = (\pi/2) - \phi_{22}, \Rightarrow \phi_{12} = (\phi + \pi/2) + \pi \end{array}\right\}$$

Using the addition identities for trigonometric functions, equation (2.4.10) can be given in terms of the single angle φ by:

$$\begin{pmatrix} C_{x'} \\ C_{y'} \\ C_{z'} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix}$$

This transformation can be viewed simple rotation of the coordinate system about the Z-axis through an angle φ. Thus, as a

$$\text{Det} \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2\phi + \sin^2\phi = +1$$

In general, every Cartesian coordinate system may be rotated around one of its primary axes in terms of a matrix whose components can be represented in terms of the rotation angle. Because these transformations revolve around a single coordinate axis, the components along that axis stay unaltered. The rotation matrices for each of the three axes are listed below:

$$\left.\begin{array}{l} \mathbf{P}_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \\ \\ \mathbf{P}_y(\phi) = \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \\ \\ \mathbf{P}_z(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}\right\}$$

Because the elements of the unit matrix are not altered by the transformation, the row and column of the matrix corresponding to the rotation axis always include the elements of the unit matrix. The cosine of the rotation angle is always present in the diagonal elements, whereas the sine of the angle modulo a sign is always present in the off-diagonal elements. For rotations around the X- or Z-axes, the top right off diagonal element has a positive value and the other has a negative sign. For rotations around the Y-axis, the situation is simply reversed. These rotation matrices are so crucial that it's worth memorizing their form so they don't have to be re-derived every time they're required. It is conceivable to demonstrate that a sequence of three consecutive coordinate rotations may be used to go from one orthogonal coordinate system to another. As a result, a generic orthonormal transformation may always be expressed as the product of three coordinate rotations around the coordinate systems' orthogonal axes. It is vital to note that the matrix product is not commutative, hence the rotation order is critical. This conclusion is so significant that the angles utilized for such a sequence of transformations have their own name.

CONCLUSION

Our investigation then moved on to a wide range of coordinate systems, each customized to individual requirements. Cartesian coordinates are the foundation of mathematical modeling and engineering due to their simplicity and orthogonal axes. Polar coordinates provide an alternate viewpoint, which is especially useful for situations requiring circular symmetry. Spherical coordinates expand our knowledge into three-dimensional space, which is necessary for astronomy, geodesy, and navigation applications. Coordinate transformations emerged as a key component of our investigation, demonstrating their importance in effortlessly converting data across multiple coordinate systems. These transformations are more than just mathematical operations; they are also bridges that promote multidisciplinary cooperation. They allow data from several sectors to cohabit and inform one another in a harmonic manner, whether in scientific study, technical design, or geographical mapping. As we come to the conclusion of our voyage, we would want to remind readers of the continuing importance of coordinate systems and transformations. These notions serve as a bridge between mathematics and reality, offering a common vocabulary for expressing and comprehending spatial connections. They support a plethora of scientific discoveries, technological advancements, and daily uses, ranging from accurate modeling of physical events to worldwide navigation. The study of coordinate systems and transformations is still an important part of our multidimensional knowledge of the cosmos.

REFERENCES:

[1] B. Liu, F. Zhang, X. Qu, and X. Shi, "A rapid coordinate transformation method applied in industrial robot calibration based on characteristic line coincidence," *Sensors (Switzerland)*, 2016, doi: 10.3390/s16020239.

[2] G. R. Mohammadi, A. G. Moghaddam, and R. Mohammadkhani, "Coordinate transformations and matter waves cloaking," *Phys. Lett. Sect. A Gen. At. Solid State Phys.*, 2016, doi: 10.1016/j.physleta.2016.01.017.

[3] Z. L. Yang, J. W. Zhang, and Y. Wang, "Dynamic Inhomogeneous Isoparametric Element with Coordinate Transformation," *J. Mech.*, 2016, doi: 10.1017/jmech.2015.99.

[4] Y. Wu, J. Liu, and H. Y. Ge, "Comparison of Total Least Squares and Least Squares for Four-and Seven-parameter Model Coordinate Transformation," *J. Appl. Geod.*, 2016, doi: 10.1515/jag-2016-0015.

[5] S. J. Ye *et al.*, “Developing a reversible rapid coordinate transformation model for the cylindrical projection,” *Comput. Geosci.*, 2016, doi: 10.1016/j.cageo.2016.01.007.

[6] W. Sima, B. Zhu, T. Yuan, Q. Yang, and J. Wang, “Finite-Element Model of the Grounding Electrode Impulse Characteristics in a Complex Soil Structure Based on Geometric Coordinate Transformation,” *IEEE Trans. Power Deliv.*, 2016, doi: 10.1109/TPWRD.2015.2408618.

[7] Y. Fang, B. Xu, and C. Sun, “Research on the relative coordinate transformation and relative position detection system of 3-DOF spherical actuator,” *Int. J. Smart Sens. Intell. Syst.*, 2016, doi: 10.21307/ijssis-2017-936.

[8] X. W. Yan, Y. R. Wang, and X. H. Wang, “A coordinate transformation based power decoupling and restriction control strategy for microgrid,” *Hunan Daxue Xuebao/Journal Hunan Univ. Nat. Sci.*, 2016.

[9] J. M. Restrepo-Flórez and M. Maldovan, “Rational design of mass diffusion metamaterial concentrators basedon coordinate transformations,” *J. Appl. Phys.*, 2016, doi: 10.1063/1.4961315.

[10] F. Fonseca, “A solution of the three dimensional navier stokes equations,” *Int. J. Pure Appl. Math.*, 2016, doi: 10.12732/ijpam.v106i1.6.

CHAPTER 12

FUNDAMENTAL CONCEPT OF ASTRONOMICAL TRIANGLE

Alok Kumar Gahlot, Assistant Professor
Department of Engineering, Teerthanker Mahaveer University, Moradabad, Uttar Pradesh, India
Email Id- dr.alokgahlot02@gmail.com

ABSTRACT:

The Astronomical Triangle is a fundamental concept in the field of astronomy, serving as the keystone for celestial navigation and positional astronomy. In this enlightening chapter, we embark on a journey through the intricacies of the Astronomical Triangle, exploring its origins, elements, and profound importance in determining the positions of celestial objects. We begin by defining the key components of the trianglezenith, celestial body, and observer's locationand the angles and distances involved. As we delve deeper, we uncover the celestial applications of this geometric construct, from measuring angles to calculating distances and pinpointing celestial coordinates. By the chapter's end, readers will have gained a deep understanding of the Astronomical Triangle's pivotal role in celestial navigation, enabling them to navigate the night sky and chart the course of stars, planets, and beyond we've embarked on a journey through the Astronomical Triangle, a cornerstone of celestial navigation and positional astronomy. Our exploration began by defining the key elements of the Astronomical Trianglethe zenith, representing the observer's position on Earth; a celestial body, such as a star or planet; and the observer's line of sight to the celestial body. We illustrated how these elements come together to form a unique triangle, one that encapsulates the observer's relationship with celestial objects.

KEYWORDS:

Angle, Astronomical, Celestial, Coordinates, Geometry, Line of Sight.

INTRODUCTION

Leonard Euler demonstrated that when one point is maintained stationary, the general motion of a rigid body corresponds to a sequence of three rotations around three orthogonal coordinate axes. Regrettably, the definition of Eulerian angles in the literature is not always consistent. We will utilize Goldstein's definitions and usually adhere to them throughout this book. The rotations are listed in the following sequence. One starts by rotating around the Z-axis. Then there's a spin around the new X-axis. This is followed by a rotation around the resultant Z"-axis. The three successive rotation angles are$[\varphi,\theta,\psi]$.Each of these rotational transformations is represented by a transformation matrix of the kind specified in equation, completing the set of Eulerian transformation matrices [1], [2],

$$\left.\begin{aligned}
\mathbf{P}_Z(\phi) &= \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\mathbf{P}_{X'}(\theta) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \\
\mathbf{P}_{Z''}(\psi) &= \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}\right\}$$

and the complete single matrix that describes these transformations is,

$$\mathbf{A}(\phi,\theta,\psi) = \mathbf{P}_{z''}(\psi)\mathbf{P}_{x'}(\theta)\mathbf{P}_{z}(\phi)$$

Thus, the components of any vector X can be found in any other coordinate system as the components of X' from,

$$\vec{X}' = \mathbf{A}\vec{X}$$

Since the inverse of orthonormal transformations has such a simple form, the inverse of the operation can easily be found from,

$$\vec{X} = \mathbf{A}^{-1}\vec{X}' = \mathbf{A}^{\mathbf{T}}\vec{X}' = [\mathbf{P}_{z}^{\mathbf{T}}(\phi)\mathbf{P}_{x'}^{\mathbf{T}}(\theta)\mathbf{P}_{z''}^{\mathbf{T}}(\psi)]\,\vec{X}'$$

The Astronomical Triangle

The rotational transformations presented in the preceding section allow for easy and quick representations of one Cartesian system's vector components in terms of those of another. The majority of astronomical coordinate systems, on the other hand, are spherical coordinate systems in which coordinates are measured in arc lengths and angles. The transfer from one coordinate frame to another is more subtle. One of the basic issues in astronomy is linking the defining coordinates of any place in the sky (say, a star or planet) to the observer's local coordinates at any given moment. This is often performed using the Astronomical Triangle, which uses a spherical triangle to connect one system of coordinates to another. Ex cathedra, the solution of the triangle is frequently described as arising from spherical trigonometry. Instead, we will demonstrate how the result (and many other outcomes) may be created using the rotational transformations that we have just explained. Due to the rotation of the earth, the celestial sphere revolves around the north celestial pole, and a great circle between the north celestial pole and the object (a meridian) seems to move across the sky with the object. At the pole, that meridian will form an angle with the observer's local prime meridian (the great circle between the north celestial pole and the observer's zenith). This angle is known as the local hour angle, and it may be determined using the right ascension and sidereal time of the object. This latter figure is derived from the observer's longitude and local time (including date). Thus, given the local time, the observer's position on the earth's surface (i.e. latitude and longitude), and the object's coordinates (i.e. Right Ascension and declination), two sides and an included angle of the spherical triangle may be deemed known. The remaining two angles and the included side must then be found. This gives the local azimuth A, the zenith distance z, which is the complement of the altitude, and the parallactic angle. While this latter amount is not required for roughly finding the object in the sky, it is important for compensating for atmospheric refraction, which causes the picture to be somewhat displaced along the vertical circle from its real position. This is then used to adjust for air extinction and is therefore beneficial for photometry [3], [4]. We will solve a distinct issue in order to solve this one. Consider a Cartesian coordinate system with a z-axis pointing along the radius vector from the origins of both astronomical coordinate systems (equatorial and alt- azimuth) to point Q. Assume the y-axis is located in the meridian plane containing Q and is oriented toward the north celestial pole. After then, the x-axis will simply be orthogonal to the y- and z-axes. Consider the vector components in this coordinate system. We may determine the components of that vector in any other coordinate frame using rotational transformations. Consider a sequence of rotational transformations that would take us through the sides and angles of the astronomical triangle and bring us back to the original xyz coordinate system. Because the transformations must precisely recreate the components of the original arbitrary vector, the transformation matrix must be the unit matrix with members ij. If we go from Q to the north celestial pole and then to the zenith, the rotational transformations will only include values relating to the specified section of our issue [i.e. (/2-), h, (/2-)]. The three local quantities [A, (/2-H),] will be involved in completing the journey from the zenith to Q. The overall transformation matrix will thus consist of six rotational matrices, the first three of which include specified angles and the final three of which involve unknowns, and this total matrix equals the unit matrix. Because each transformation matrix represents an orthonormal transformation, its inverse is just its transpose. As a result, we may build a matrix equation in which one side includes matrices of known variables and the other side has matrices of unknown quantities [5], [6]. Let's follow this program and see where it takes us. The angle will represent the first rotation of our basic coordinate system [-(π/2 - δ)]. This will take us through the complement of the declination and align the z-axis with the earth's rotating axis. Because the rotation will be around the x-axis, the rotation matrix from equation (2.4.14) will be used.

$$\mathbf{P}_x\left[-\left(\frac{\pi}{2}-\delta\right)\right]=\begin{pmatrix}1 & 0 & 0\\ 0 & \sin\delta & -\cos\delta\\ 0 & \cos\delta & \sin\delta\end{pmatrix}$$

Now, counterclockwise or positively rotate (h) around the new z-axis that is aligned with the polar axis, such that the new y-axis is on the local prime meridian plane heading away from the zenith. The hour angle is included in the rotation matrix for this transformation, thus,

$$\mathbf{P}_z(\mathrm{h})=\begin{pmatrix}\cos h & \sin h & 0\\ -\sin h & \cos h & 0\\ 0 & 0 & 1\end{pmatrix}$$

Continue the journey by rotating via [+(π/2 - φ)] until the coordinate system's z-axis coincides with a radius vector through the zenith. This will need a positive rotation around the x-axis in order to get the required transformation matrix,

$$\mathbf{P}_x\left[-\left(\frac{\pi}{2}-\phi\right)\right]=\begin{pmatrix}1 & 0 & 0\\ 0 & \sin\phi & \cos\phi\\ 0 & -\cos\phi & \sin\phi\end{pmatrix}$$

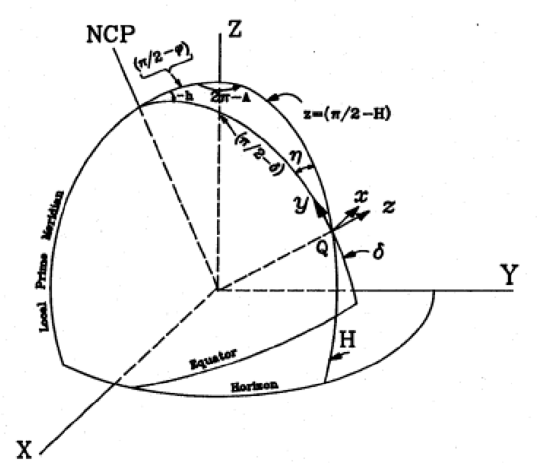


Figure 1: Shows the Astronomical Triangle with the zenith in the Z-direction.

This triangle must be solved in order to perform transformations between the Alt-Azimuth coordinate system and the Right Ascension-Declination coordinate system.

The hour angle h and the distance from the North Celestial Pole are used to calculate the latter coordinates. Rotate around the z-axis via the azimuth [2π-A] such that the y-axis now points toward the point in question Q.

This is another z-rotation to get the correct transformation matrix [7], [8].

$$\mathbf{P}_z[2\pi - A] = \begin{pmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We may restore the z-axis to its previous position by performing a negative rotation around the x-axis via the zenith distance (π/2-H), resulting in a transformation matrix,

$$\mathbf{P}_x\left[-\left(\frac{\pi}{2} - H\right)\right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin H & -\cos H \\ 0 & \cos H & \sin H \end{pmatrix}$$

Finally, the coordinate frame may be aligned with the initial frame by rotating the z-axis via an angle [π+η] , resulting in the final transformation matrix,

$$\mathbf{P}_z[\pi + \eta] = \begin{pmatrix} -\cos\eta & -\sin\eta & 0 \\ +\sin\eta & -\cos\eta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad .$$

Because the final consequence of all of these transformations is to return to the original coordinate frame, the product of all transformations produces the identity matrix or

$$\mathbf{P}_z[+(\pi+\eta)]\mathbf{P}_x(-z)\mathbf{P}_z(2\pi - A)\mathbf{P}_x[+(\pi/2 - \phi)]\mathbf{P}_z(h)\mathbf{P}_x[\delta - \pi/2] = \mathbf{1}$$

We may separate the knowns from the unknowns by remembering that the inverse of an orthonormal transformation matrix is its transpose so that,

$$\mathbf{P}_z[+(\pi+\eta)]\mathbf{P}_x(-z)\mathbf{P}_z(2\pi - A) = \mathbf{P}_x^{\mathbf{T}}(\delta - \pi/2)\mathbf{P}_z^{\mathbf{T}}(h)\mathbf{P}_x^{\mathbf{T}}[(\pi/2) - \phi]$$

The matrix products suggested by equation (2.6.8) must now be explicitly performed, and the nine elements on the left-hand side must match the nine elements on the right-hand side. These nine connections give all of the spherical triangle's available relations in a natural manner [9], [10].

These, of course, include the standard relations cited for the astronomical triangle solution. These nine connections are:

$$\left.\begin{aligned}
&\sin H = \cos h \cos\phi \cos\delta + \sin\delta \sin\phi \\
&\cos H \cos A = \sin\delta \cos\phi - \cos h \cos\delta \sin\phi \\
&\cos H \sin A = -\cos\delta \sin h \\
&\cos H \cos\eta = \cos h \sin\delta \cos\phi - \cos\delta \sin\phi \\
&\cos H \sin\eta = \sin h \cos\phi \\
&\sin A \sin\eta + \sin H \cos A \cos\eta = -(\cos\delta \cos\phi + \cos h \sin\phi \sin\delta) \\
&\sin A \cos\eta + \sin H \cos A \sin\eta = -\sin\phi \sin h \\
&\cos A \sin\eta + \sin H \sin A \cos\eta = +\sin\delta \sin h \\
&\cos A \cos\eta + \sin H \sin A \sin\eta = -\cos h
\end{aligned}\right\}.$$

Because the altitude is specified as being in the first or fourth quadrants, only the first of these relations uniquely identifies H. The next two will then supply the azimuth A in a unique manner, and the following two will allow for the unique definition of the parallactic angle. As a result, these relations are sufficient to convert coordinates from the defining coordinate frame to the observer's frame or vice versa. The more typical solution to the astronomical triangle, on the other hand, may be derived from.

$$\mathbf{P}_z[+(\pi+\eta)]\mathbf{P}_x(H-\pi/2)\mathbf{P}_z(2\pi-A)\mathbf{P}_x[(\pi/2)-\phi] = \mathbf{P}_x^{\mathbf{T}}(\delta-\pi/2)\mathbf{P}_z^{\mathbf{T}}(h)$$

where only the last row of the matrices is considered. These elements yield,

$$\left.\begin{aligned}
&\sin h \cos\delta = -\sin A \cos H \\
&\cos h \cos\delta = -\cos A \cos H \sin\phi + \sin H \cos\phi \\
&\sin\delta = +\cos A \cos H \cos\phi + \sin H \sin\phi
\end{aligned}\right\}$$

Because we determined the azimuth from the north point, our findings vary from those given in several astronomy textbooks. To get the usual results, we must substitute A with (π-A). After discussing how to detect objects in the sky in various coordinate frames and how to link those frames, we will now move on to a short explanation of how to locate them in time.

Time

Time is the independent variable in Newtonian mechanics, and we have talked nothing about it so far. Newton considered time to be absolute and evenly 'flowing' over all space. Albert Einstein's creation of what became known as the Special Theory of Relativity in 1905 demonstrated that this intuitively obvious idea was erroneous. However, for objects traveling in the solar system, the challenges posed by special relativity are often minor. The less nuanced understanding of how time is measured complicates the concept of time. Historical changes, like other convoluted conceptions of science, have helped to greatly confuse the definition of what should be a simple idea. We shall refer to time units as seconds, minutes, hours, days, years, and centuries (there are more, but we will disregard them for the purposes of this work). These components' interactions are complicated and have been shaped by history. In a broad sense, time may be defined as the interval between two occurrences. The challenge occurs when deciding which events should be picked for everyone to utilize. To put it another way, what "clock" will we use to define time? Because clocks respond to physical forces, we are faced with the technical challenge of determining the best accurate clock. Clocks that measure the interval between atomic processes and have an accuracy of 1 part in

1011 to 1 part in 1015 are currently the most accurate. Clocks like this serve as the foundation for measuring time, and the time they keep is known as international atomic time (TAl for short). However, clocks that replicate the rising and setting of the sun or the rotation of the planet have been used to keep time for ages.

Certainly, primitive man recognized that not all days were equal in duration and so could not be used to define a unit of time. The gap between two consecutive transits (crossings of the local meridian) of the sun, on the other hand, is virtually constant. If the earth's orbit were exactly round, the sun's journey along the ecliptic would be uniform in time. As a result, it could not be consistent around the equator. Because of the non-uniformity of motion over the equator, consecutive transits of the sun will vary. To make things worse, the earth's orbit is elliptical, thus motion along the ecliptic is not even uniform. One may compensate for this by keeping time by the stars. Sidereal time refers to time that is related to the apparent motion of the stars, and local sidereal time is important to astronomers because it identifies the position of the origin of the Right Ascension-Declination coordinate frame as viewed by a local observer. As a result, it decides where objects are in the sky. Local sidereal time is generally defined as the observer's hour angle of the vernal equinox. However, when our capacity to precisely measure time intervals increased, it became evident that the globe did not spin at a constant pace. While a rotating item seems to be the ideal clock, other objects working via those forces generate abnormalities in the spin rate since it seems to be independent of all natural forces. In truth, the earth makes a terrible timepiece.

Not only does the rotation rate fluctuate, but the position of the junction of the north polar axis with the earth's surface moves somewhat during the year. Furthermore, long-term precession caused by torques created by the sun and moon acting on the earth's equatorial bulge causes the polar axis, and hence the vernal equinox, to shift in its position among the stars. This, in turn, affects the time gap between subsequent transits of any particular star. Time scales based on the rotation of the earth do not conform to Newton's concept of evenly flowing time. As a result, we need another sort of time, a dynamical time adequate for describing the solutions to Newtonian equations of motion for solar system objects. This time is known as terrestrial dynamical time (TDT), and it is an extension of what was previously known as ephemeris time (ET), which was discontinued in 1984. Because it is to be Newton's smoothly flowing time, it may be directly connected to atomic time (TAl) with an additional constant to achieve agreement with the historical ephemeris time of 1984. As a result, we have:

$$\text{TDT} = \text{TAI} + 32.184 \text{ seconds}$$

Unfortunately, we and the atomic clocks are on a moving body with a gravitational field, and both of these qualities will impact the pace at which clocks operate when compared to equivalent clocks positioned in an inertial frame devoid of gravity and accelerative motion. To establish a time suitable for spacecraft navigation in the solar system, we must account for the effects of special and general relativity and determine an inertial coordinate frame in which to keep track of time. The origin of such a system may be assumed to be the solar system's barycenter (center of mass), and we can define barycentric dynamical Time (TDB) as that time. Because the relativistic components are so tiny, the difference between TDT and TDB is less than.002 sec. The Astronomical Almanac3 has a particular formula for calculating it.Terrestrial dynamical time is the time used to compute the velocity of solar system objects. However, it is only about right for viewers attempting to find things in the sky. Another time scale that compensates for the earth's uneven rotation is required for this.

Historically, such time was referred to as Greenwich Mean Time, but this word has now been replaced with the more grandiose-sounding universal time (UT). The basic form of universal time (UT1) is used to calculate civil time standards and is based on star transits. As a result, it is tied to Greenwich mean sidereal time and has non-uniformities caused by fluctuations in the earth's rotation rate. This is required to locate an object in the sky. The Astronomical Almanac gives the difference between universal time and terrestrial dynamical time, which now (1988) amounts to approximately one full minute since the earth is "running slow." Of course, the difference between the dynamical time of theory and the observable time imposed by the earth's rotation must be determined after the fact, but previous behavior is utilized to estimate the present. Finally, coordinated universal time (UTC) is the time that acts as the global time standard and is broadcast by WWV and other radio stations.

$$|\mathrm{UT1} - \mathrm{UTC}| < 1 \text{ second}$$

Coordinated universal time runs at the same pace as atomic time (give or take relativistic adjustments), but is modified by an integral number of seconds to stay close to UT1. This adjustment may occur up to twice a year (on December 31 and June 30), resulting in a systematic discrepancy between UTC and TAl. In 1972, the difference was 10 seconds. From then until now (1988), adjustments totaling 14 seconds have been required to ensure near agreement between the sky and the ground. Coordinated universal time is near enough to UTl to find objects in the sky, and it may be efficiently converted to local sidereal time in lieu of UTl by scaling by the sidereal to solar day ratio. Because the International Astronomical Union has decided that terrestrial Longitude should be defined as growing positively to the east, local mean solar time will only be.

$$(\mathrm{LMST}) = \mathrm{UTC} + \lambda$$

where A is the longitude of the observer. The same will hold true for sidereal time so that,

$$(\mathrm{LST}) = (\mathrm{GST}) + \lambda$$

where the Greenwich sidereal time (GST) can be obtained from UTl and the date. The local sidereal time is just the local hour angle of the vernal equinox by definition so that the hour angle of an object is,

$$h = (\mathrm{LST}) - (\mathrm{R.A.})$$

We have chosen rising hour angles measured west of the prime meridian. We can measure the mobility of objects in the solar system (TDT) and determine their position in the sky (UTl and LST) using the proper time scale. There are several minor adjustments, such as the earth's barycentric motion (its motion around the center of mass of the earth-moon system) and so on. All of these adjustments are essential and form a study in and of itself for people interested in time to better than a millisecond. Knowledge of the local sidereal time as established by UTC will often adequate for the straightforward acquisition of astronomical objects through a telescope.

CONCLUSION

It found the Astronomical Triangle's practical uses in celestial navigation. We discovered how it enables us to estimate angles such as the altitude and azimuth of celestial objects, which provides vital information for celestial observation and orientation. The Astronomical Triangle is also used to calculate distances to celestial bodies such as the Moon using techniques such as lunar parallax. It is a necessary instrument for obtaining celestial coordinates, assisting astronomers and navigators in estimating the locations of stars, planets, and other celestial phenomena. As we complete our voyage, we ask readers to consider the Astronomical Triangle's enormous importance. It is a geometric structure that connects Earth to the universe, enabling humans to track celestial objects and explore the night sky. The Astronomical Triangle is a timeless tool that creates a deeper connection to the celestial marvels above and leads us in our study of the cosmos, whether you're an amateur astronomer looking at the stars or a navigator plotting your voyage over the seas.

REFERENCES:

[1] M. U. Saleem, "Gnomon Assessment for Geographic Coordinate, Solar Horizontal & Equatorial Coordinates, Time of Local Sunrise, Noon, Sunset, Direction of Qibla, Size of Earth & Sun for Lahore Pakistan," *Open J. Appl. Sci.*, 2016, doi: 10.4236/ojapps.2016.62011.

[2] I. Šprajc, "Astronomy and power in Mesoamerica," in *Astronomy and Power: How Worlds Are Structured Proceedings of the SEAC 2010 Conference* , 2016.

[3] V. Katz (book editor) and C. Montelle (review author), "The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook," *Aestimatio Crit. Rev. Hist. Sci.*, 2015, doi: 10.33137/aestimatio.v4i0.25818.

[4] Z. W. Li, T. Zhang, N. Zhang, and M. G. Sun, "High precision orientation of faint space objects," *Guangxue Jingmi Gongcheng/Optics Precis. Eng.*, 2015, doi: 10.3788/OPE.20152309.2627.

[5] C. Brinch and C. P. Dullemond, "Interferometer predictions with triangulated images: Solving the multiscale problem," *Mon. Not. R. Astron. Soc.*, 2014, doi: 10.1093/mnras/stu524.

[6] L. Zhang, X. He, Z. H. Wei, and J. M. Guo, "Fast celestial positioning for space objects based on star identification," *Guangxue Jingmi Gongcheng/Optics Precis. Eng.*, 2014, doi: 10.3788/OPE.20142211.3074.

[7] L. L. Yu, J. Ji, and S. Wang, "Shape, thermal and surface properties determination of a candidate spacecraft target asteroid (175706) 1996 FG3," *Mon. Not. R. Astron. Soc.*, 2014, doi: 10.1093/mnras/stu164.

[8] F. J. T. Salazar, E. E. N. Macau, and O. C. Winter, "Alternative transfer to the Earth-Moon Lagrangian points L4 and L5 using lunar gravity assist," *Adv. Sp. Res.*, 2014, doi: 10.1016/j.asr.2013.11.055.

[9] J. M. Steele, "A forgotten discipline," *Metascience*, 2014, doi: 10.1007/s11016-013-9836-9.

[10] A. Mohamed, "History of Surveying & Mapping in Malaysia," *FIG Congr.*, 2014.