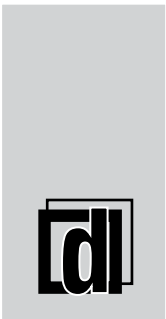


A TEXTBOOK OF MECHANICS

**H.D. PANDE
S.N. SINGH
R.N. LAL
NEERAJ KAUSHIK**





A Textbook of Mechanics

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Knowledge is Our Business

A TEXTBOOK OF MECHANICS

By H.D. Pande, S.N. Singh, R.N. Lal, Neeraj Kaushik

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CHAPTER 1

STRATEGIES FOR SOLVING PROBLEMS

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ABSTRACT:

Solving problems is a big part of physics. You will run into issues whether you are reading a book on a well-known topic or doing cutting-edge research. It is reasonable to conclude that, in the latter scenario (which is currently relevant given what you have in your possession), the capacity to solve issues with the subject at hand serves as the genuine litmus test of comprehension. While reading about a subject is often a required but by no means sufficient stage in the learning process. The more time you can spend problem-solving, which is inherently an active effort, in addition to reading, which is often a more passive activity, the better. As a result, I've included a ton of problems and exercises in this book. But if I'm going to provide you with all of these issues, I need to at least provide you with some broad approaches to handling them. The focus of this chapter is on these tactics. When handling a challenge, you want to keep these ideas in mind at all times. Naturally, they are often insufficient on their own; you won't get very far without comprehending the physical principles underlying the relevant topic. However, they may greatly ease your life if you combine them with your physical knowledge.

KEYWORDS:

Equations, Issue, Strategies, Time, Units.

INTRODUCTION

You should use a variety of broad techniques without reluctance while resolving an issue. As follows: Make care to explicitly name all the relevant quantities (forces, lengths, masses, etc.) in the diagram. In certain sorts of situations, diagrams are vitally essential. Drawing a diagram may transform a seemingly impossible issue into one that is very straightforward, as in the cases of "free-body" diagram problems or relativistic kinematic difficulties. Diagrams are always incredibly useful, even when they aren't absolutely necessary. Without a doubt, a picture is worth a thousand words (and maybe even more if you name things!). You can unknowingly simply do this in your brain while solving a straightforward task. However, writing things down directly is tremendously helpful when solving more complex situations. For instance, if you're attempting to locate three unknowns but have only recorded two facts, you may start looking for the third one as you know you must be missing it (assuming the issue can be solved). A conservation law, an equation using $F = ma$, etc[1], [2].

Find symbolic solutions

If the supplied quantities are described numerically in a problem you are solving, you should immediately convert the numbers to letters and formulate your solution in terms of the letters. After getting a letter-based response, you may enter real number values to get a numerical response. The use of letters has a lot of benefits: Quicker, indeed. A g and a 4 are significantly simpler to multiply when written down next to one another on paper than when multiplied together on a calculator. And if you used the latter approach, you would surely need to use your calculator a few times while working on a problem.

It's less likely that you will make a mistake. In a calculator, it's relatively simple to input an 8 instead of a 9, but it's unlikely that you'll accidentally write a q instead of a g on a piece of paper. However, if you do, you'll immediately see that it has to be a g. Since no one has provided you with the value of q, you won't simply give up on the issue and declare it unsolvable and forever. On the several questions that have been provided, you may see the general impact of your response. You can see, for instance, that it increases with quantities a and b, decreases with quantity c, and is independent of quantity d. A symbolic response has a far larger amount of information than a numerical one. Additionally, symbolic solutions often have a beautiful, attractive appearance. Its and special cases are checkable these checks complement the prior "general dependence" benefit. However, because of their significance, we will put off talking about them and instead focus on them. Having said all of this, it should be recognised that dealing with letters might sometimes result in some confusion. For instance, it could be difficult to solve a system of three equations with three unknowns unless you punch in the actual values. But working just with letters is incredibly favorable in the great majority of issues[3], [4].

Verify special/limiting instances

If the answer is a numerical one, check the order of magnitude. If you come up with a precise numerical solution to an issue, be sure to do a sanity check to determine the number's validity. If you estimated the length of time a vehicle would slide down the ground before coming to rest and your result was a km or a millimetre, you know you made a mistake. These kind of mistakes often result from forgetting powers of ten (such as when converting from kilometres to metres) or multiplying something instead of dividing it.

Whether it be a physics issue or not, you will surely get across situations when you don't get a solid solution, either because the computation is too difficult or you don't feel like performing it. However, under these circumstances, a reasonable prediction, to the closest power of 10, may typically still be made. For instance, if you pass a structure and wonder how many bricks it contains or how much labour went into creating it, you can probably respond with an acceptable response without having to do any complex calculations. Enrico Fermi, a physicist, was renowned for his fast estimation skills and for producing order-of-magnitude estimates with little to no math. Thus, an issue where the only objective is to get the closest power-of-10 approximation is referred described as a "Fermi problem." Of course, there are occasions in life when you need to be more accurate than the closest power of 10.

DISCUSSION

Like the well-known Olympic ten rings, the 100 states, the ten-date weeks, and the birds that all fly with the same set of wings. We'll go through highly crucial checking unit techniques and unique instances in the two sections that follow. The method of numerical problem solving will then be covered which is what you need to do when you are left with a set of equations that you are unable to solve. In contrast which are applicable to almost every situation you'll encounter, solving equations numerically is something you'll only perform sometimes. As a result, isn't completely equivalent to those sections. But each and every physics student should be aware of it. We'll use different conclusions drawn from the book's later chapters in all three of these parts. Don't worry at all about the physics underlying these conclusions since they are absolutely irrelevant for the purposes of the current discussion.

There will be plenty of time later for that! Learning what to do with a problem's outcome after you've acquired it is the key goal here[5], [6].

Dimensional analysis and units

The powers of mass, length, and time attached to a quantity are its units, or dimensions. The units of speed, for instance, are length per time. There are two key advantages to taking units into account. Prior to beginning a problem, looking at units may help you determine the general shape of the solution, up to numerical factors. Second, you may determine if your result has a probability of being true by checking units at the conclusion of a computation, which is something you should always do. While it won't say that your response is unquestionably accurate, it may say that it is unquestionably wrong. For instance, you would realise it's time to review your work if your aim in solving a problem was to discover a length but you came up with a mass instead.

Your units are incorrect, the instructor yelled. "What a feature! Your church weights six joules. And those inside are eight gauss distant from the preacher and four hours wide. You will typically employ the second of the aforementioned two advantages in real life. However, because they may be a bit more interesting, let's give a couple instances that relate to the first benefit. We would need to use the conclusions drawn from subsequent chapters in order to precisely answer the three situations below. But first, let's explore how far we can get using dimensional analysis alone. We'll denote the units with the symbol "[]" and use the abbreviations M for mass, L for length, and T for time. You may work out this by noticing that Gm_1m_2/r has the dimensions of force, which in turn has dimensions ML/T^2 , from $F = ma$. We'll write a speed as $[v] = L/T$ and the gravitational constant as $[G] = L^3/(MT^2)$. As an alternative, you may just substitute kg, m, and s for M, L, and T, respectively has units that are $1/T$. Only $g/4$ is a combination of the dimensionful values we are given that has units of $1/T$. The most generic form of the frequency, however, is since we are unable to completely rule out any θ dependency. Here, f is a dimensionless function of the dimensionless variable just so happens to be about equal to 1 for minor oscillations, hence the $g/4$ is about equivalent to frequency. However, it is impossible to demonstrate this using merely dimensional analysis; the actual solution to the issue must be found. The higher-order terms in the expansion of f become significant for bigger values of θ which deals with the leading correction. There is only one mass in the issue, hence m/M cannot be a factor in the frequency (measured in units of $1/T$). If it did, the units of mass wouldn't be cancelled, leaving just pure inverse-time. We said before that the only combination of the dimensionful quantities we provided.

It is $g/4$ and has units of $1/T$. This is simple to observe, but the next approach will always be effective for more challenging issues where the right mix is less clear. The supplied dimensionful quantities should be combined to form a general product, and the units of this product should then be expressed in terms of a, b, and c. In order to get units of $1/T$ here, we must first: When the powers of the three different types of units on each side of this equation are matched, it produces. What can be said about the pendulum's overall energy (measured in terms of potential energy relative to the lowest point)? Energy will be covered, but for now, all we need to know is that it is measured in ML^2/T^2 units. Only may be combined with the provided dimensionful constants in this way. The energy must have the form, where f is a function, since we can't completely rule out any θ dependency. We have reached the limit of dimensional analysis. The total energy is equal to the potential energy at the greatest point, if we really use a little science. We can show that by using the Taylor expansion for $\cos$ (see

Appendix A for a description of Taylor series). In light of this, the maximum angle θ plays a crucial influence in the energy, in contrast to the frequency result above.

Rough estimates and limiting instances

However, similar to testing units, it could inform you that your answer is unquestionably inaccurate rather than saying that it is certainly right. In general, your intuition about limiting circumstances will be far more accurate than your intuition about general parameter values. This fact ought to work in your favor[7], [8].

Let's go through a few instances of the second advantage. Just accept them for the time being. The beginning phrases provided in each of the examples below are derived from different examples throughout the book. I'll mostly restate what I'll say later, when we really go through the issues, here.

The Taylor series approximations are a technique that is often used in limiting case analysis; the series for various functions are provided in Appendix A. 1.4 numerically solving differential equations addressing differential equations is a common step in the process of addressing physics problems. A differential equation is one in which the variable you're attempting to solve for has derivatives, often with regard to time in our physics issues. The Lagrangian method, which we'll cover, $F = ma$, and/or $\Sigma \tau = I\alpha$, or some other method, always results in a differential equation.

Although some issues generate intricate differential equations, you will eventually come into one that you can't quite solve (either because it's really difficult to solve, or because you can't come up with the right solution). After accepting that you won't know the precise answer, you should consider how to acquire a good estimate of it. Fortunately, it's simple to create a quick programme that will provide you with a precise numerical solution to your issue. If the system isn't chaotic, which won't be a problem for the systems we'll be working with, you can get any required precision with adequate computing time. By using a common issue that we'll address precisely and in-depth we'll illustrate the process. With $\omega = \sqrt{k/m}$, this is the equation for a mass on a spring. In, we'll see that the answer may also be expressed as $x(t) = A \cos(\omega t + \phi)$. (1.14) But let's act as though we are unaware of this. It would appear that if someone were to come along and give us the values we should be able to obtain $x(t)$ and $\dot{x}(t)$ for all subsequent t solely by utilising In essence, we should know everything about the system if we know how it begins and how it develops, as determined. Thus, this is how we determine $x(t)$ and $\dot{x}(t)$.

The idea is to discretize time into short intervals, or units, and then work out what happens at each succeeding moment in time. If we are aware of both $x(t)$ and $\dot{x}(t)$, we may use the definition of $\dot{x}$ to quickly determine (roughly) the value of x at a later time. Similarly, if we know $\dot{x}(t)$ and $\ddot{x}(t)$, we can simply use the definition of $\ddot{x}$ to discover (roughly) the value of $\dot{x}$ at a later time.

The relations are easily defined using the derivative definitions. Both $x(t + \Delta t)$ and $\dot{x}(t + \Delta t)$ result in $x(t) + \dot{x}(t)\Delta t$, respectively. These two equations enable us to advance through time, acquiring consecutive values for x , $\dot{x}$, and $\ddot{x}$. They work in conjunction with which provides us with x in terms of $\ddot{x}$. Here is an example of a typical programme. (Though this is a Maple programme, the main notion should be obvious even if you aren't.) Imagine that of course, there is also the definitional formula for $\ddot{x}$, which uses the third derivative. However, this would need understanding of the third derivative, and so on with higher derivatives, leading to an endless chain of relationships. A motion equation, such as which in general might be a $F = ma$, $\Sigma \tau = I\alpha$, or

Euler-Lagrange equation, ties x back to x (and potentially $\dot{x}$), producing a tangled relationship between x , $\dot{x}$, and $\ddot{x}$ while obviating the necessity for an endless and pointless chain. Because computation time isn't a consideration in our uncomplicated system, we wrote the programme as simply as possible without giving it any thought. However, creating a programme that is as efficient as feasible is a key component of the problem-solving process when dealing with more complicated systems that call for programmes for which computing time is a concern[9], [10].

The whole Taylor series with higher-order terms is approximated by these equations in first order. In other words, there are ambiguities in how the programme may be built, making it impossible for the aforementioned technique to be 100% accurate. Which comes first, line 7 or line 5? That is, should we utilise the x at time t or $t + \Delta t$ in order to calculate x at time $t + \Delta t$? And after line 6, should line 7 come? The idea is that the order doesn't really matter for extremely tiny Δt . Additionally, the order is completely irrelevant in the limit $\Delta t \rightarrow 0$. Although this is a fantastic technique, it shouldn't be overused. If all else fails, it's comforting to know that we can always get a respectable numerical estimate. However, because this enables us to see the system's general behaviour, we should focus our early efforts on deriving the right algebraic equation. In addition, nothing surpasses the truth. Nowadays, people tend to depend a little bit too much on computers and calculators without taking the time to consider the true nature of an issue. The ability to do maths on a page has fallen to shockingly low levels. On Mathematical, quadratical equations are solved, and we don't know our age on birthdays[11], [12].

A lunar pendulum

Escape velocity - The escape velocity on a planet's surface is provided by the formula $v = \sqrt{2GM/R}$ where M and R are the planet's mass and radius, respectively, and G is Newton's gravitational constant. (Velocity is the escape velocity. When air resistance is disregarded, the adage "What goes up must come down" is disproved.)

(a) Instead of writing v as M , write it as the average mass density.

(a) Given that the radius of Jupiter is 11 times larger than the radius of the earth and that the average density of the earth is four times that of Jupiter, what is v_J/v_E Projectile moving downhill * A hill slopes downward at an angle to the horizontal. A m -gram projectile is launched perpendicular to the slope at v_0 speed. Let the velocity establish an angle with the horizontal when it ultimately settles on the slope. The angle depends on which of the variables, m , v_0 , and g ? waves on a string what effect do the string's mass M , length L , and tension (or force) T have on the wave speed? Consider a vibrating water drop whose frequency ν is dependent on its radius R , mass density, and surface tension S . Surface tension is measured in units of (force)/ (length). How are R and S related to?

Approximations and limiting instances

Atwood's apparatus-Take into consideration the three masses and three frictionless pulleys "Atwood's" machine. The acceleration of m_1 may be shown to be given by:

Cone frustum - A cone frustum has a base radius of b , a top radius of a , and a height of h . assuming that the volume is one of the following numbers

CONCLUSION

The examination of limiting instances, or maybe we should say special cases, has similar advantages to the consideration of units. It may first aid in the beginning stages of a condition. Imagine making a specific length extremely huge or very little, for example, and then observe what happens to the behavior if you're having problems understanding how a certain system acts. It will be simpler to comprehend how the length affects the system generally once you have persuaded yourself that it does in fact affect the system in extreme cases (or perhaps you will find that it has no effect at all). This will make it simpler to write down the necessary quantitative equations (conservation laws, $F = ma$ equations, etc.), which will enable you to fully solve the problem. In other words, changing the different parameters and monitoring how it affects the system may provide a wealth of knowledge. Second, much as with verifying units, you should always check limiting cases (also known as special instances) at the conclusion of a computation.

REFERENCES (10-15 Ref.)

- [1] P. Intaros, M. Inprasitha, and N. Srisawadi, "Students' Problem Solving Strategies in Problem Solving-mathematics Classroom," *Procedia - Soc. Behav. Sci.*, 2014.
- [2] C. Jiang, S. Hwang, and J. Cai, "Chinese and Singaporean sixth-grade students' strategies for solving problems about speed," *Educ. Stud. Math.*, 2014.
- [3] A. Youssef-Shalala, P. Ayres, C. Schubert, and J. Sweller, "Using a general problem-solving strategy to promote transfer," *J. Exp. Psychol. Appl.*, 2014.
- [4] N. Abdullah, L. Halim, and E. Zakaria, "VStops: A thinking strategy and visual representation approach in mathematical word problem solving toward enhancing STEM literacy," *Eurasia J. Math. Sci. Technol. Educ.*, 2014.
- [5] M. Montague, J. Krawec, C. Enders, and S. Dietz, "The effects of cognitive strategy instruction on math problem solving of middle-school students of varying ability," *J. Educ. Psychol.*, 2014.
- [6] E. Atteh, E. A. Andam, W. Obeng-Denteh, C. A. Okpoti, and A. Johnson, "The Problem Solving Strategy of Solving Mathematical Problem: The Case Study of Esaase Bontefufuo Senior High Technical School, Amansie West District of Ghana," *Int. J. Appl. Sci. Math.*, 2014.
- [7] I. S. Robert, "Investigating the problem-solving strategies of revisers through triangulation," *Transl. Interpret. Stud.*, 2014.
- [8] R. Anvari, Z. JianFu, and S. H. Chermahini, "Effective Strategy for Solving Voluntary Turnover Problem among Employees," *Procedia - Soc. Behav. Sci.*, 2014.
- [9] E. Sagone and M. Elvira De Caroli, "A Correlational Study on Dispositional Resilience, Psychological Well-being, and Coping Strategies in University Students," *Am. J. Educ. Res.*, 2014.
- [10] K. Borna and V. H. Hashemi, "An Improved Genetic Algorithm with a Local Optimization Strategy and an Extra Mutation Level for Solving Traveling Salesman Problem," *Int. J. Comput. Sci. Eng. Inf. Technol.*, 2014.
- [11] S. Nesmachnow, "An overview of metaheuristics: accurate and efficient methods for optimisation," *Int. J. Metaheuristics*, 2014.
- [12] H. L. Swanson, A. Moran, C. Lussier, and W. Fung, "The effect of explicit and direct generative strategy training and working memory on word problem-solving accuracy in children at risk for math difficulties," *Learn. Disabil. Q.*, 2014.

CHAPTER 2

THE CONCEPT OF STATICS

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ABSTRACT:

A static configuration is one in which nothing is moving. Newton's second law, $F = ma$, which we'll go over in great depth in the next chapter, states that the total external force acting on an item must be zero if it doesn't move. Of course, the opposite is not true. If an object travels at a constant nonzero velocity, the total external force acting on it is likewise zero. However, we'll limit our discussion to statics issues. Finding out what the different forces must be in order for each item to experience zero net force (and zero net torque, but that's the subject) is the main objective of statics problems. This objective entails decomposing the force into its constituent parts as a force is a vector. Polar coordinates, Cartesian coordinates, or another set are all options. Usually, the issue makes it evident which system will facilitate your computations. Once you've decided on a system, all you have to do is insist that there is zero external force acting overall in any direction. The majority of the many diverse kinds of forces that exist in the world are the large-scale consequences of complex processes occurring at lower scales. For instance, the chemical bonds that keep the molecules of a rope together are electrical forces, and they are what cause the tension in the rope.

KEYWORDS:

Force, Friction, Rope, Stick, Tension

INTRODUCTION

In other works, the topic of statics often comes up later, following the chapters on force and torque. However, compared to what we'll be doing later in this book, the usage of force and torque in statics issues is rather minor. Therefore, I'll explain here the absolute minimum of force and torque ideas required for statics issues since we won't be using much of the equipment that we'll be creating later. This will cause us to face a new set of issues. The fundamental ideas of statics are straightforward to describe, yet statics issues may be very challenging. So be sure to work on a lot of them to ensure that you comprehend everything [1], [2].

Coordinating forces

There is no need to examine every aspect of the forces operating at the molecular level while solving a rope-related mechanical issue. Simply calling the troops in Give the rope a "tension" and focus on the issue. Four different forces are often mentioned:

Tension

The force that a rope, stick, etc. produces as it is pulled is known as tension. Except for the end points, which experience stress on one side and a force on the other side from whatever item is tied to the end, every component of the rope experiences tension in both directions. The strain along the rope may sometimes change. This is best shown by the example of "Rope wrapped around a pole" at the conclusion of this section. In other situations, the level of stress must be uniform throughout. The tension must be the same throughout a massless

rope, for instance, or a massless rope hanging over a frictionless pulley. If it were not, there would be a net force acting on at least a portion of the rope, and $F = ma$ would result in an infinite acceleration for this (massless) piece [3], [4].

Standard Force

This is the force that an item is subjected to from a surface that is perpendicular to it. The normal force plus the friction force combine to make up the total force exerted by a surface (see below). However, there is merely the typical force when it comes to slippery or icy surfaces that have no friction. The surface really experiences a little amount of compression, acting as a very stiff spring and producing the normal force. The surface is compressed until the restoring force is equal to the force required to prevent the item from being compressed any further.

The direction of the force is essentially the sole distinction between a "tension" and a "normal force" in most situations. A spring may be used to mimic either circumstance. When there is tension, the spring (which might be a rope, a stick, or anything else) is stretched and the force acting on the item is instead directed towards the spring. When a normal force is applied, the spring is compressed and the force is applied to the target item in an opposite direction to the spring. Sticks, for example, may produce both ordinary forces and tensions. However, a rope, for instance, finds it difficult to exert a normal force. In reality, a compressive force is more often referred to as a "compressive tension," or a "negative tension," when applied to long items like sticks. According to these definitions, a tension might therefore be neutral. Whatever the case, it's only semantics. Any of these descriptions of a compressed stick will be understood by others [5], [6].

Friction

Friction is the force that a surface exerts to an item that is parallel to it. Some surfaces have a lot of friction, like sandpaper. Some surfaces, like oily ones, have almost little friction. The terms "kinetic" friction and "static" friction refer to the two different forms of friction. In this chapter, we won't discuss kinetic friction, which is friction between two moving objects. Saying that the kinetic friction between two things is typically a fair approximation equivalent to the usual force that exists between them. The "coefficient of kinetic friction" or "k" is the proportionality constant, and it depends on the two surfaces in question. F thus equals kN , where N is the normal force. The force acts in the opposite direction to the motion. Two objects at rest in relation to one another are the subject of static friction. $F \leq sN$ (where s is the "coefficient of static friction") is the equation for the static situation. The inequality indication is visible. All we can say is that the static friction force has a maximum value equal to $F_{\max} = sN$ before we solve a problem. It is probably less in a particular issue. For instance, if you push a block of significant mass M that is resting on a surface with friction coefficient s just enough to prevent it from moving to the right, the friction force will obviously not be equal to $sN = sMg$ to the left. The block would be propelled to the left by such a force. The actual friction force is just the reverse of the minuscule force you apply and equal to it [7], [8].

Gravity

Consider two point objects that are R distance apart and have masses M and m . The force between these objects is attracting, according to Newton's gravitational force equation, with a magnitude of $F = GMm/R^2$ and a gravitational constant of $G = 6.67 \cdot 10^{-11} \text{ m}^3/(\text{kg s}^2)$. The same rule holds true for spheres with nonzero sizes as well, as we'll demonstrate in Chapter 5.

To put it another way, the centre of a sphere may be thought of as a point mass. As a result, an item on Earth's surface experiences gravitational pull.

where R is the earth's radius and M is the earth's mass. G is defined by this equation. You can see that after plugging in the numbers, we get $g = 9.8 \text{ m/s}^2$. Every item on the earth's surface experiences a downward force of mg (g fluctuates somewhat throughout the earth's surface, but let's disregard this). If the item is not speeding, then other forces—such as gravity, normal forces, etc.—must exist in order to balance the overall force at zero. The spring force described by Hooke's law, $F = kx$, is another ubiquitous force.

DISCUSSION

Adjusting the torques

In a statics issue, we must additionally balance torques in addition to forces. Chapters 8 and 9 will provide a lot more information about torque, but we'll need F_1 first. Here's an essential fact. Think about the scenario in Fig. in which three forces are F_2 applied perpendicular to a stick that is thought to be stationary. F_1 a b F_3 is the inner force, and F_2 are the forces at the ends. We do, of course, because the stick is at rest, $F_3 = F_1 + F_2$. But we also have the relationship that follows:

Suspender rope

From one end, a rope hung vertically has length L and mass density per unit length. Determine the tension along the rope as a function of height.

Block in an aircraft

On a plane that is inclined at an angle, a block is placed. Assume that the friction force is sufficient to maintain the block's resting position. What are the frictional and normal forces operating on the block's horizontal components? What is the greatest value of these horizontal components?

Chain not moving

A frictionless tube is in the form of an arbitrary function that has its ends at the same height and is located in the vertical plane. As shown in Fig., a chain with a constant mass per unit length is positioned end to end within the tube. 2.9. Establish that the chain is stationary by taking into account the net force of gravity along the curve.

Keeping a journal

In front of a vertical wall is a book of mass M . Friction between the book and the wall has a coefficient of. As shown in Fig., you want to prevent the book from falling by applying a force F on it at an angle $(\theta/2)$ to the horizontal. 2.10.

(a) What is the minimal F necessary for the given?

What is this minimal F 's lowest value for? What is the equivalent minimum F ?

(c) What is the minimum value of below which a F cannot exist that maintains the book?

Rope in the air

On a plane that is inclined at an angle, a rope with length L and mass density per unit length is located (see Fig. 2.11). The rope's upper end is fastened to the plane by nails, and there is a

coefficient of friction between them. What are the potential values for the rope's peak tension?

Sustaining a Disc

(a) As shown in Fig., a massless string supports a disc of mass M and radius R . 2.12. The disk's surface has no friction. What is the string's tension? What is the usual force that the string typically exerts to the disc per unit length?

(b) Allow friction with coefficient to now exist between the disc and the string. What is the string's lowest point's least potential tension?

Circles between objects

The placement of each of the following planar items. Between two circles of equal radius R , at 2.13. a unit's mass density. Each object's area is and its contact points' radii form an angled at a 45° angle to the horizontal. Find the horizontal force that acts in each scenario applied to the circles in order to keep them together. What is the maximum or lowest force for this object? An isosceles triangle with a common side length of L is example, An L -height rectangle, A circle

Chain for hanging

A chain between them hangs with a constant mass density per unit length two fixed spots on two different walls. Find the chain's overall form. The function defining the shape should have one unknown constant in addition to an arbitrary additive constant. The length l of the chain, the vertical distance between the support points, and the horizontal distance d between the walls all affect the unknown constant in your solution Find an equation that uses these supplied values and the unknown constant to determine it [9], [10].

Softly Dangling

A chain with a constant mass density per unit length is suspended between two supports that are spaced $2d$ apart and at the same height. What chain length should be used to reduce the amount of force acting on the supports? The fact that a hanging chain has the formula $y(x) = (1/x) \cosh(x)$ may be used. You'll ultimately need to use a calculator to answer an equation.

Climber of mountains

The goal of a mountaineer is to ascend a frictionless conical mountain. He intends to do this by lassoing (throw a rope over the top with a loop) and climbing up the rope. Assume that the climber is very short, causing the rope to hang down the mountain as seen in Fig. 2.16. There are two shops at the base of the mountain. One vendor offers "cheap" lassos (a rope segment connected to a loop with a predetermined length; see Fig. 2.17. The other offers "deluxe" lassos, which are composed of a single piece of rope with an adjustable loop whose length may alter without the rope coming into contact with itself. The conical mountain has an angle at its summit when seen from the side. What angles can the mountain climber reach if he employs a "cheap" lasso? A "deluxe" lance? (Hint: The response in the "cheap" scenario isn't 90°).

The broadest instance.

Think about a stick with a length of l with pressures F applied upward at the ends and $2F$ applied downward at the middle (see Fig. 2.18). The stick cannot translate or spin because of

symmetry or because there is no net force acting on it. The stick's left end may be thought of as a pivot if we so want. We can see that a force F applied at a distance of 4 from a pivot is equivalent to a force $2F$ applied at a distance of $4/2$ from the pivot in that they both have the same effect of cancelling out the rotational effect of the downward $2F$ force. If we then remove the force F on the right end and replace it with a force $2F$ at the middle, then the two $2F$ forces in the middle cancel, and the stick remains at rest.

Now think about the scenario in which pressures F are applied downward at the $4/3$ and $24/3$ marks and upward at the ends (see Fig. 2.19). The stick cannot translate or spin because of symmetry or because there is no net force acting on it. Think of the stick's left end as its pivot. According to the above sentence, a force F at $24/3$ is equal to a force $2F$ at $4/3$. We now have a total force of $3F$ at the $4/3$ point after making this change. As a result, we can observe that an applied force of F at a distance of 4 is comparable to an applied force of $3F$ at a distance of $4/3$. Your next step is to demonstrate by using induction to establish that a force F applied at a distance 4 is comparable to a force nF applied at a distance $4/n$.

The tension's direction

Demonstrate that the tension runs the whole length of a massless or gigantic rope that is totally flexible.

Locate the force

Supports at either end of a stick with mass M lift the stick with a force of $Mg/2$. Now place a second support in the centre, say a distance of b from the first support and a distance of a from the second; see Fig. 2.20. What forces are currently provided by the three supports? Is this a solution?

Moving sticks

As shown in Fig., one stick rests on another. 2.21. They meet at a right angle, and the right stick creates an angle with the horizontal. The right stick's end is infinitesimally far from the left stick. The two sticks have a coefficient of friction of μ . The sticks are both hinged at the ground and have the same mass density per unit length. What is the smallest angle at which the sticks are not thrown?

Providing ladder support

The bottom end of a ladder with mass M and length L is pivotally linked to the ground. It is supported by a massless stick of length 4 that is at an angle with the horizontal and is pivotally connected to the ground (see Fig. 2.22). The stick and the ladder are parallel to one another. Determine the force the stick is applying to the ladder.

Keeping the stick in check

Determine the relationship between density and position for a semi-infinite stick (one that extends to infinity in one direction) to have the following property: If the stick is cut at any point, the remaining semi-infinite piece will balance on a support that is placed 4 from the end (see Fig. 2.23).

The thread

A spool is made up of an outer circle with a radius of R that rolls on the ground and an axle with radius r . A thread is wound around the axle and pulled at an angle θ with the horizontal

with tension T . What should be in order to prevent the spool from spinning, given R and r ? Assume that there is adequate friction between the spool and the ground to prevent slipping. What is the highest value of T for which the spool stays at rest, given R , r , and the coefficient of friction between the spool and the ground? What should r be in order to make the spool slide from the static position with the smallest feasible T , given R and? In other words, what value of r should be chosen to minimise the upper constraint on T in section (b)? What is the value of T as a result?

On a circle stick

A circle of radius R supports a stick with mass density per unit length at its centre. The stick's top end is tangent to the circle and forms an angle with the horizontal. Assume that friction occurs at all points of contact and that it is sufficient to maintain the system at rest. Discover the circle's friction with the earth [11], [12].

Sticks and circles leaning

As shown in Fig., a great deal of sticks (with mass density per unit length) and circles (with radius R) lean on one another. 2.26. Each stick is tangent to the one before it and creates an angle with the horizontal a circle at its top. Each stick has a hinge connecting it to the ground. In contrast to the prior issue, the other surface is frictionless. In the maximum what is the usual force between a stick and the circle it rests on, very far to the right, among a very large number of sticks and circles? Assume that the last circle is immobile since it is leaning against a wall.

CONCLUSION

We have learned numerous significant things from our research on static equilibrium in structural engineering. The core ideas of statics have been confirmed, notably the idea that stable structures need balanced pressures and moments. The examination of different structural components showed how important load distribution and shape are for preserving equilibrium. Our research also highlighted the need of precisely estimating responses to guarantee the security of structural systems. We have emphasised the value of meticulous analysis and mathematical modelling in structural engineering via this study. Static equilibrium principles must serve as the cornerstone of engineers' and designers' work since any departure from equilibrium might result in structural collapse and jeopardize public safety. Our results further highlight the need of continuing study and development in the statics sector. A thorough grasp of equilibrium becomes more and more important as structures grow more demanding and complicated. To improve structural designs and guarantee their stability under varied situations, future research may investigate cutting-edge computational techniques and materials.

REFERENCES:

- [1] J. Lubliner and P. Papadopoulos, *Introduction to solid mechanics: An integrated approach*. 2014.
- [2] "Engineering mechanics 1: statics," *Choice Rev. Online*, 2014.

- [3] I. Grattan-Guinness, “From anomaly to fundament: Louis Poincaré’s theories of the couple in mechanics,” *Hist. Math.*, 2014.
- [4] C. Papadopoulos, A. I. Santiago-Román, and G. Portela-Gauthier, “Initial results of introducing design and simulation based instruction in mechanics of materials,” in *ASEE Annual Conference and Exposition, Conference Proceedings*, 2014.
- [5] S. Hollerer, “Numerical validation of a concurrent atomistic-continuum multiscale method and its application to the buckling analysis of carbon nanotubes,” *Comput. Methods Appl. Mech. Eng.*, 2014.
- [6] W. S. Rone and P. Ben-Tzvi, “Mechanics modeling of multisegment rod-driven continuum robots,” *J. Mech. Robot.*, 2014.
- [7] Q. Yang, E. Biyikli, and A. C. To, “Multiresolution molecular mechanics: Convergence and error structure analysis,” *Comput. Methods Appl. Mech. Eng.*, 2014.
- [8] J. Burkhardt, “Performance of at-risk students in dynamics following a statics class with additional recitation time,” *International Journal of Mechanical Engineering Education*. 2014.
- [9] K. Van Canneyt and P. Verdonck, “Mechanics of Biofluids in Living Body,” in *Comprehensive Biomedical Physics*, 2014.
- [10] K. M. Steele, S. R. Brunhaver, and S. D. Sheppard, “Feedback from in-class worksheets and discussion improves performance on the statics concept inventory,” *Int. J. Eng. Educ.*, 2014.
- [11] S. A. Walton and T. Boothby, “What is straight cannot fall: Gothic architecture, Scholasticism, and dynamics,” *Hist. Sci.*, 2014.
- [12] H. Moseley, *The Mechanical Principles of Engineering and Architecture*. 2014.

CHAPTER 3

THE CONCEPT OF THE NEWTON LAWS

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ABSTRACT:

This research examines how Newton's Laws of Motion might be used in many real-world situations and their ongoing value. Newton's Laws have been crucial to our understanding of how things move and interact with forces, and their applicability goes beyond basic physics to include engineering and astronomy. We have investigated the three laws of motion in many situations, such as motion on Earth, celestial bodies, and mechanical systems, using a mix of theoretical analysis and experimental observations. Our study demonstrates the importance of these rules in forming our contemporary technological environment in addition to reaffirming their universality. This research emphasizes the significance of Newton's Laws as a cornerstone of physics instruction and engineering practice by demonstrating their broad application.

KEYWORDS

Celestial, Force, Law, Mass, Particle

INTRODUCTION

Finding out what occurs to a certain collection of things in a specific physical setting is the overarching purpose of classical mechanics. We need to understand the causes of the items' motions in order to solve this puzzle. There are primarily two approaches to doing this assignment. The first one includes Newton's laws, which you are likely acquainted with. The current chapter's topic is this. The Lagrangian approach is the second, more sophisticated option. It should be emphasised that each of these approaches yields the same results in the end, making them equally suitable for tackling any issue. But they are founded on quite different ideas [1], [2] .

The Newtonian laws

In his Principia Mathematical, Newton presented his three laws in 1687. Although I think it's debatable to ascribe the descriptor "intuitive" to a collection of claims that wasn't formally codified until only 300 years ago, these rules certainly seem to make sense. The laws might be described as follows, at least.

The first rule states that unless a force is applied, a body travels at a constant speed, which might be zero.

The force exerted on a body is equal to the temporal rate of change of its momentum, according to the second law.

Third law: There is an equal and opposite force acting on another body for every force acting on one body.

The extent to which these claims are definitions and the extent to which they are physical laws might be debated for days on end. The first law, according to Sir Arthur Eddington, essentially states that "every particle continues in its state of rest or uniform motion in a

straight line except insofar as it does"; however, there is more to the three laws than Eddington's unflattering remark suggests. Let's examine each one in turn.

A disclaimer: My opinion on which portions of the laws are definitions and which portions contain substance is expressed in this section. But you need to treat everything here with a grain of salt. See Anderson (1990), Keller (1987), O'Sullivan (1980), and Eisenbud (1958) for more reading

First rule

This law defines zero force, among other things. It also defines an inertial frame, which is a frame of reference in which the first rule is valid. Since the word "velocity" is used, we must specify the frame with regard to which we are measuring the velocity. In a random frame, the first rule does not hold true. An inertial frame is one that travels with constant speed, therefore it fails in the case of a revolving turntable, for instance. However, this is unclear since we need to specify what the frame is travelling with constant speed in relation to. Putting all of this aside, an inertial frame is a unique kind of frame that upholds the first law. There isn't much physical material here; instead, we have two definitions of "force" and "inertial frame" that are interwoven. The fact that the rule applies to all particles is crucial, however. Therefore, all free particles travel at the same speed if we have a frame in which one free particle does so. This is a content-filled statement. We cannot have a group of free particles travelling at constant speed while another particle does an elaborate jig [3], [4].

Second rule

The definition of momentum³ is mv . The second rule states that if m is constant $F = ma$ in which a dv/dt . Only an inertial frame—which is what the first law defines—is subject to this rule. Observe what the first law accomplishes best for objects that are free to move or at rest. The crucial frame is described. By name, "inertial", where the expression of the second rule is found. There is more to the second law than just defining force, despite what you would believe. The law makes a subtle suggestion that this "force" has an existence apart from the particle whose " m " is referenced in the law (more on this in the third law below). For instance, a spring force is completely independent of the particle it operates on. Additionally, the gravitational force, GMm/r^2 , is dependent on both the particle and another mass. If we add the so-called "fictitious" forces, it is feasible to change the situation such that Newton's laws are valid in this frame [5], [6].

Of course, nothing we do here is relativistically based. The relativistic adaptation of the mv equation is provided in Chapter 12. In this chapter, we'll assume that m is constant. But don't worry, Chapter 5 will give us plenty of practise shifting mass (in rockets and other things). If you want to create some additional definitions, you may specify the amount $G = m^2a$. You actually can't go wrong when you define something, provided that you haven't previously defined the amount to be something else. This is absolutely legal to do. This definition, however, is entirely meaningless. The key is that the definitions don't relate to one another; you may define it for any acceleration and for any particle in the universe. Simply said, there is no (uncontrived) quantity in the universe that, when "acting" on masses m and $2m$, produces accelerations in the ratio of 4:1. The only thing the quantity G has to do with is the particle for which you defined it. The second law's basic assertion is that there is a quantity F that, when acting on several particles, yields the same value of ma . The assertion that such a

thing exists is much more than a definition. In keeping with this, observe that the second law states that $F = ma$ and not,

$F = mv$ or $F = m \frac{dx}{dt}$, for instance. These utterances violate both the first rule and reality, in addition to being contradictory. Contrary to the first rule, $F = mv$ would state that a nonzero velocity needs a force.

DISCUSSION

Additionally, $F = m \frac{dx}{dt}$ would contradict the first rule by stating that a particle travels with constant acceleration (instead of constant velocity) until pushed upon by a force. It is crucial to understand that the second rule applies to all particles, just as the first law did. In other words, if two particles with masses m_1 and m_2 are subjected to the same force then, their velocities are connected by regardless matter what the common factor is, this relationship remains valid. We can thus predict what the ratio of their a 's will be when they are exposed to any other force after we have used one force to determine the relative masses of two objects. Naturally, we still haven't fully defined mass provides an experimental procedure for calculating an object's mass in terms of a reference mass, such as 1 kg. Simply comparing its acceleration to the standard mass's when both are subjected to the same force will do [7], [8].

Third rule

This rule states, among other things, that when two isolated particles interact by some force, their accelerations are inversely proportional to their masses and opposite in direction. Alternatively, the third law basically asserts that an isolated system's overall momentum is preserved and time-independent. Consider two particles that interact exclusively with one another and nothing else in the cosmos in order to observe this. Next, we have where the forces acting on m_1 and m_2 , respectively, are denoted by F_1 and F_2 . This proves that Newton's third law ($F_1 = -F_2$) and momentum conservation ($\frac{dp_{\text{total}}}{dt} = 0$) are identical. The same logic applies when there are more than two particles, but we'll keep this more general example, along with a lot of other features of momentum, for Chapter 5.

This is a rule of pure content since there isn't much that can still be specified by it. It's really not always true, thus it can't be a definition. It works for "pushing" and "pulling" forces but not, for instance, the magnetic force. In such instance, momentum is lost in the electromagnetic field, resulting in the conservation of the overall momentum of the particles and the field. Fields, however, are not the topic at hand. Particles only. Therefore, the third rule will always apply in whatever circumstance that we consider.

One very significant fact is included in the third law. It claims that unless another particle is also accelerating someplace else, we will never locate a particle accelerating. Even if the other particle may be far away, as in the case of the earth-sun system, it is always present. Note that if we had only the second law, it would be entirely possible for a given particle to accelerate on its own without anything else in the universe happening, provided that a particle with a similar mass but half the mass accelerated with half the acceleration when placed in the same location, etc. If the second law were to apply, everything would be good. If everything were consistent, we might argue that a force operating at the spot had a certain value. However, according to the third rule, this is simply not how the world—at least the one in which we live—operates. A force without a counterpart has the appearance of being magical in a way, but a force with an equal and opposing counterpart has a "cause and effect" character that seems (and is seemingly) more physical.

In the end, though, we shouldn't give Newton's laws too much weight since, while being a tremendous intellectual accomplishment and functioning well for commonplace physics, they are the principles of an approximation. Newtonian physics is a limiting instance of quantum mechanics and relativity, which are themselves limiting examples of far more accurate theories. Fundamental interactions between particles (or waves, or strings, or whatever) have absolutely nothing to do with what we think of as forces.

Free-body illustrations

The second law is the one that enables us to use numbers. $F = ma$ may be used to determine the acceleration given a force. And if we are given the starting location and velocity of an item, we may predict its behaviour (i.e., its position and velocity) by knowing the acceleration. There are two fundamental sorts of scenarios that often emerge, however this procedure may sometimes be labor-intensive. In many situations, you are just given a physical scenario (like as a block lying on a plane, threads connecting masses, etc.), and it is your responsibility to use $F = ma$ to determine all the forces operating on every item. Since the forces often point in different directions, it is simple to lose sight of them. Therefore, it is advantageous to separate the objects and identify all the forces that are at work on each of them. The current section's topic is this. In other cases, you are required to solve the $F = ma$ equation (we'll simply deal with one dimension here) since the force is supplied directly as a function of time, location, or velocity. It may be difficult or perhaps impossible to precisely solve certain differential equations. Let's have a look at the first of these two situations, in which we are presented with a real-world scenario and need to identify all the factors at play. The phrase "free-body diagram" refers to a diagram that depicts all the forces acting on a certain item. We just list all the $F = ma$ equations that are implied by each such diagram for each item in the setup. We may then solve the resulting system of linear equations in different unknown forces and accelerations. An example will help you to better understand this process.

Mass M_1 is supported on a plane with an inclination angle of θ , and mass M_2 dangles over the side. A massless thread that passes through a massless pulley joins the two masses. M_1 and the plane have a coefficient of kinetic friction of μ . M_1 is awakened from sleep. What is the acceleration of the masses assuming that M_2 is big enough to drag M_1 up the plane? What is the string's tension? Draw all the forces on the two masses as the first step in the solution. FIG. displays them. The tension and gravity are the forces acting on M_2 . Gravity, friction, tension, and the normal force are the forces acting on M_1 . The friction force is pointed out here. We assume that M_1 travels up the aircraft and down the plane.

We can now write out all of the $F = ma$ equations after drawing all of the forces. We might divide M_1 into horizontal and vertical components, but it is much simpler to utilise the components parallel and perpendicular to the plane of interest where we have taken use of the fact that the two masses accelerate in the same direction, with downward specified as the positive direction for M_2 . We have also taken use of the fact that the tension on the string is the same at both ends because if it were not, there would be a net force acting on a portion of the string, which would then experience infinite acceleration due to its masslessness.

In, there are four unknowns but just three equations (T , a , N , and f). Since we are assuming that M_1 is really moving, we get a fourth equation, $f = \mu N$, which allows us to utilise the formula for kinetic friction. This results in the second equation above having the value $f = \mu M_1 g \cos \theta$. The initial equation then changes to $T = M_1 g \sin \theta + \mu M_1 g \cos \theta$. We only have an after adding this to the third equation, therefore we discover

We need $M_2 \geq M_1(\sin \theta)$ for M_1 to really accelerate downhill (that is, $a > 0$). Since the system began at rest, the range of M_2 for which it doesn't accelerate (rather, it merely stays there) is if M_2 is very tiny, M_2 must almost exactly equal $M_1 \sin \theta$ for the system to be static. Equation (3.7) also suggests that even if $M_2 = 0$, M_1 won't slide down if $\tan \theta \leq \mu$. It's obvious what you should choose as the objects you're going to apply forces on in situations like the one above. However, in other issues, there are one of these two coordinate systems often performs substantially better than the other when working with inclined planes. Sometimes it's unclear which one to use, but if one system starts to malfunction, you can always switch to the other. You have a variety of subsystems to choose from, but you must be cautious to consider all the pertinent influences on each subsystem. What amounts you're looking for will determine which subsystems you should choose.

Atwood's device

A fixed support is suspended from a massless pulley. Over the pulley, a massless thread connecting the two masses m_1 and m_2 hangs (see Fig. 3.11). Find the string's tension as well as the masses' acceleration. Double Atwood's machine depicts a twin Atwood's machine with masses m_1 , m_2 , and m_3 . Find the masses' accelerations.

Atwood's infinite machine

Think about Fig. 3.13's endless Atwood's machine. Each pulley is covered by a string that has one end tied to a mass and the other end to another pulley. All pulleys and threads are massless, and all masses are equal to m . After being repaired, the masses are simultaneously released. What is the top mass's acceleration? This infinite system may be defined as follows. Think of it as having N pulleys, where the $(N + 1)$ th pulley would have been replaced with a nonzero mass. Take N as the limit in such case.) Line of pulleys $\times N + 2$ (3.4) As shown in Fig. 3.14, equal masses swing from a set of pulleys. What are the velocities of every mass?

Ring of pulleys

Take into consideration the pulley arrangement in Fig. 3.15. The string, which is a loop with no ends, is suspended from a ring's bottom by N fixed pulleys. N pulleys suspended on a string are connected to N masses, $m_1, m_2, \dots, m_N$. What are the velocities of every mass?

Descending a plane

- (a) A block descends a frictionless plane that is inclined at an angle θ starting at rest. What has to be in order for the block to go a certain horizontal distance in the shortest period of time?
- (b) The same question, with the addition that the block and the plane now have a coefficient of kinetic friction.

Sliding to the side when flying

On a plane that is inclined at an angle θ , a block is put. The block and the plane have a coefficient of friction $\mu = \tan \theta$. A kick is applied to the block, causing it to travel first horizontally down the plane at speed V (i.e., perpendicular to the direction heading directly down the plane). How quickly does the block move after a very long time?

Moving aero plane

On a frictionless plane of mass M and angle of inclination, a block of mass m is kept immobile. The aircraft is supported by a flat, frictionless surface. The restriction is lifted. What is the plane's horizontal acceleration?

Exponential force

A force $F(t) = ma_0 e^{bt}$ is applied to a particle of mass m . Position and speed are both at their starting points. Locate $x(t)$. The force $F(x) = kx$, where $k > 0$, acts on a particle with mass m . X_0 is the beginning location, while 0 is the initial speed. Locate $x(t)$.

Falling chain

A chain of length 4 is stretched out horizontally on a frictionless table, and a length y_0 is suspended from a hole in the table below. The link is cut loose. Find the length that hangs through the hole as a function of time (don't worry about t after the chain loses touch with the table).

Launching a beach ball

An upward beach ball is launched with starting velocity v_0 . Assume that $F_d = mv$ represents the air's drag force. What speed was the ball travelling at just before it impacted the ground, v_f ? (It suffices to have an implicit equation.) If the ball were thrown in a vacuum, would it spend longer or less time in the air?

Holding a pencil in balance

Think of a pencil that stands on its tip erect before falling over. Imagine the pencil to be a mass m at the end of a massless rod that is 4.20 in length. Assume that the pencil's initial angular speed is 0 and that it first produces the angle θ with the vertical. What is the angle as a function of time when it is small (so that $\sin \theta \approx \theta$)? The angle will ultimately grow to be huge. You would believe that by making the initial θ and $\dot{\theta}$ sufficiently tiny, it should be able (at least theoretically) to make the pencil balance for an infinite amount of time. Heisenberg's uncertainty principle, which limits how accurately we can determine the location and momentum of an object, turns out to be the cause of this. A pencil cannot be balanced over an extended period of time (like a particle). It all comes down to the fact that you can't be certain the pencil is originally both at the top and at rest. To be quantitative about this is the aim of the challenge. You're likely to be surprised by the time restriction [9], [10].

Without delving into quantum physics, let's simply say that the uncertainty principle states that $\Delta x \Delta p \geq \hbar/2$, where $\hbar = 1.05 \cdot 10^{-34}$ J s is Planck's constant. This is true up to factors of order 1. This has fairly ambiguous implications, so we'll simply interpret it as indicating that the beginning circumstances fulfil $(\Delta x)(\Delta p) \approx \hbar$. Your objective is to determine how long it will take your (t) solution in section (a) to reach order 1 under this restriction. In other words, estimate the longest duration the pencil can remain balanced.

Motion of the projectile

Maximum trajectory area

A ball is thrown at speed v from ground level at a height of zero. What angle should it be thrown at to maximise the area under the trajectory?

Bouncing ball

A ball is hurled directly up, reaching a height of h . It continuously drops and bounces. It returns to a certain f -fraction of its prior height after each bounce. Calculate the total time and distance travelled before coming to rest. What is the typical speed?

Perpendicular velocities

Demonstrate that the starting and final velocities are perpendicular to one another in the maximum-distance scenario in section (b) of the example.

Throwing a ball down a cliff

From the cliff's edge, a ball is hurled with velocity (v) at a height (h). What angle of inclination should it be thrown at to ensure that it covers the most horizontal distance? How far is this distance at its maximum? Assume that the ground is horizontal underneath the cliff [11], [12].

Redirected motion

When a ball is dropped at rest at height h above the ground, it bounces (without losing speed) off a surface at height y . The surface has an angle to it, causing the ball to bounce off at an angle to the horizontal. What should y and be in order for the ball to travel the greatest possible horizontal distance before it lands?

CONCLUSION

Our study of Newton's Laws of Motion concludes by highlighting their continued importance and usefulness in a wide range of fields. The law of inertia, which is part of Newton's First Law, is still a key idea in understanding how things behave whether they are at rest or moving uniformly. It serves as the foundation for comprehending how things react to outside forces and the reasons for their motion. Engineers, scientists, and inventors use Newton's Second Law, which connects force, mass, and acceleration, as a crucial tool for creating and analyzing systems for anything from equipment and automobiles to space travel. The application of this rule to the prediction and regulation of motion has revolutionized technology and enhanced our quality of life. The application of numerous machines and systems is made possible by Newton's Third Law, often known as the principle of action and response. Its uses range from the construction of effective transportation systems to biomechanics and beyond, including rocket propulsion. Our study has shown that the rules proposed by Sir Isaac Newton centuries ago still serve as fundamental instruments for comprehending the physical world and furthering technological development. The everlasting principles of Newton's Laws serve as a compass as we explore new horizons in science and engineering, allowing us to solve complicated problems and unravel the secrets of the cosmos. In conclusion, Newton's Laws of Motion continue to be essential to the study of physics and engineering because they provide a solid foundation for understanding how things behave while they are in motion. Their useful applications in real-life situations confirm their enduring worth and relevance in our constantly changing environment.

REFERENCES:

- [1] M. C. Palenik, "Quantum mechanics from Newton's second law and the canonical commutation relation $[X, P] = i$," *Eur. J. Phys.*, 2014.
- [2] S. Little and M. Little, "Laboratory test of Newtons law of gravity for small accelerations," *Class. Quantum Gravity*, 2014.
- [3] F. Mohaddes, S. Islam, R. Shanks, M. Fergusson, L. Wang, and R. Padhye, "Modification and evaluation of thermal properties of melamine-formaldehyde/ n-eicosane microcapsules for thermo-regulation applications," *Appl. Therm. Eng.*, 2014.
- [4] S. Schneiderbauer and M. Krieger, "What do the Navier-Stokes equations mean?," *Eur. J. Phys.*, 2014.
- [5] D. Rui Pires, P. .Dr. S. M.J.Ali, and A. S. A. Al-banaa, "Stress analysis on main landing gear for small aircraft," *AL-Rafdain Eng. J.*, 2014.
- [6] J. Handhika and E. Kurniadi, "Analisis Kesulitan Mahasiswa Dalam Memahami Konsep Vektor Gaya Pada Hukum Newton," *J. Penelit. LPPM IKIP PGRI Madiun*, 2014.
- [7] B. K. Temiz and A. Yavuz, "Students' misconceptions about Newton's second law in outer space," *Eur. J. Phys.*, 2014.
- [8] N. Roos, "Entropic forces in Brownian motion," *Am. J. Phys.*, 2014.
- [9] J. P. Petit and G. D'Agostini, "An alternative model, explaining the VLS, created by the gravitational interaction of two matter populations, one composed of positive mass and the other of negative mass," in *Journal of Physics: Conference Series*, 2014.
- [10] M. H. Rahman, I. L. Ahmad, M. Sayahkarajy, M. R. Ahmad, M. Nakajima, and T. Fukuda, "Lab-on-chip microfluidic system for single cell mass measurement," in *IECBES 2014, Conference Proceedings - 2014 IEEE Conference on Biomedical Engineering and Sciences: "Miri, Where Engineering in Medicine and Biology and Humanity Meet,"* 2014.
- [11] J. Göémez and M. Fiolhais, "The physics of articulated toys - A jumping and rotating kangaroo," *Eur. J. Phys.*, 2014.
- [12] R. D. Saxton *et al.*, "An X-ray and UV flare from the galaxy XMMSL1 J061927.1-655311," *Astron. Astrophys.*, 2014.

CHAPTER 4

AN OVERVIEW ON THE BASICS OF OSCILLATIONS

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ABSTRACT:

When x and its temporal derivatives enter a differential equation exclusively via their first powers, the equation is said to be linear. An example an illustration of a nonlinear differential equation. The phrase homogeneous differential equation is used if the right-hand side of the equation is zero. We use the term inhomogeneous differential equation if the right-hand side is some function of t , as in the instance. This chapter teaches readers how to resolve homogeneous and inhomogeneous linear differential equations. We had best come up with a methodical approach to solve them as they keep coming up in physics. In order to master the approaches we will use, it is preferable to solve examples, therefore let's begin by resolving a few simple differential equations. In this chapter, it is assumed that x is a function of t . Consequently, a dot stands for temporal distinction.

KEYWORDS:

Differential, Equation, Mass, Motion, Solution

INTRODUCTION

Choose an exponential solution, or one with the formula $x = Ae^t$, as the second option. The result of substituting into $x = axe$ is $a =$ instantly. Consequently, the answer is $x = Ae^{at}$. Because our differential equation is homogeneous and linear in x , we should take note that we cannot solve for A (A cancels out). The starting situation dictates A . This approach could come out as ridiculous and a little unnecessary. The procedure is, nevertheless, fairly generic since, as we will show later, estimating these exponential functions (or sums of them) is really the most general thing we can do. You may be afraid that even if we have uncovered one answer using this approach, we might have overlooked another one. However, according to the general theory of differential equations, there is only one independent solution to a first-order linear equation (we'll simply accept this fact for now). Therefore, if we locate only one answer, we may be sure we've found the full puzzle [1], [2].

It will undoubtedly function, however there is a lot easier approach to use when our equation is linear in x . As in the first case above, we may predict a solution of the type and then figure out what α must be. Once again, A cancels out, so we are unable to solve for it plugging into $x = axe$ results in $a = a$. As a result, we have discovered two answers. An arbitrary linear combination of them provides the most comprehensive answer, which you can immediately verify functions properly. The beginning circumstances dictate what A and B are. You could worry that even though we have identified two solutions to the problem, we might have overlooked additional possibilities, like in the first case above. Our second-order linear problem, however, only has two independent solutions, according to the general theory of differential equations. We know that we have discovered all of the answers since we have identified two separate ones.

A monumentally significant characteristic of linear differential equations is that the addition of two separate solutions yields a solution to our problem. As you should examine (see Problem 4.1), this rule does not apply to nonlinear differential equations like $x^2 \ddot{x}$ because the process of squaring the two solutions after adding them results in a cross term that negates the equality. The superposition principle refers to this characteristic. In other words, combining two answers results in a third. In other words, superposition results from linearity. Because of this, theories controlled by linear equations are far simpler to understand than those governed by nonlinear ones. For instance, most General Relativity systems have exceedingly challenging solutions since the theory is built on nonlinear equations. You may take your answers for equations with a single major condition (linear equations).

With definite goals, then layer them on top of one another. Let's talk a little bit more about the It is useful to define an a as a 2 , where a is a real integer, if a is negative. Consequently, the answer. If required, this may be expressed in terms of trig functions using the formula. There are many formats for writing the solution: One of the aforementioned types will function more effectively than the others depending on the particulars of a given system. These expressions' numerous constants are connected to one another. For instance, the cosine sum formula results in $C = E \cos 1$ and $D = E \sin 1$. In each of the aforementioned formulas for $x(t)$, take note that there are two free parameters. The starting circumstances, such as the location and velocity at $t = 0$, dictate these parameters. The amount, in contrast to these free characteristics, is defined by the specific physical system we are working with. We'll see that, for a spring, for instance, $\omega = k/m$, where k is the spring constant. ω is unaffected by the starting circumstances.

Once again, there is a connection between the different constants. Some information on the hyperbolic trig functions is included in Appendix A in case you're not acquainted with them. Although the answer to Eq. Although (4.2) is accurate for both signs of a , it is often clearer to express the negative- a solutions in the trig forms or the explicit eit exponential form. It is impossible to overstate the value of our strategy for predicting exponential solutions. Although it may look quite constrictive, it functions. You should be persuaded of this by the examples in the next sections of this chapter. This is how we work, it's crucial. Differential for equations we solve. It completes the task, it's also a lot of fun [3], [4].

Basic harmonic movement

Now let's practise some actual physical issues. We'll begin with straightforward harmonic motion. This is the movement a particle undergoing a force of $F(x) = kx$. A mass linked to a massless spring resting on a frictionless surface is the standard mechanism for basic harmonic motion. For further information on why this is the case. A standard spring has a force of the kind $F(x) = kx$, where x is the displacement from equilibrium. This is known as "Hooke's law," and it is valid so long as the spring is not too compressed or extended. This idiom eventually loses meaning for any genuine spring. However, if we assume a kx force, $F = ma$ results in $kx = mx$, or

This trig solution demonstrates that the system oscillates back and forth in time indefinitely. The angular frequency is written as ω . The location and velocity return to their original values if t rises by 2π , which also raises the cosine's argument by 2π . $T = 2\pi/\omega = 2\pi \sqrt{m/k}$ is the period (the length of time for one full cycle). $\nu = 1/T = \omega/2\pi$ is the frequency expressed in cycles per second (Hz). The amplitude, or greatest distance the mass may travel from the origin, is the constant A (or rather its absolute value if A is negative). It should be noted that the velocity is given by the equation $v(t) = \dot{x}(t) = A\omega \sin(\omega t + \phi)$.

The starting circumstances establish the constants A and ϕ . For instance, if $x(0) = 0$ and $\dot{x}(0) = v$, then we must have $A \cos \phi = 0$ and $A \sin \phi = v$, which leads to $\phi = \pi/2$ and $A = v$ (or $\phi = 3\pi/2$ and $A = -v$, but this yields the same result). As a result, we have $x(t) = (v/\omega) \cos(\omega t + \pi/2)$. If we express it as $x(t) = (v/\omega) \sin(\omega t)$, it seems a bit neater. It seems that if the information you have is limited to the starting location and velocity, x_0 and v_0 , then the equation in Eq. (4.3) often performs the best. Another configuration that includes beginning circumstances is provided in Problem 4.3.

DISCUSSION

The simple pendulum, which is a mass that hangs on a massless string and swings in a vertical plane, is another classic system that exhibits (about) simple harmonic motion. Let l represent the string's length and θ represent the angle it forms with the vertical (see Fig. 4.2). Then, $mg \sin \theta$ is the gravitational force acting on the mass in the tangential direction. $F = ma$ thus yields in the tangential direction the radial $F = ma$ equation just provides us with the tension, which we won't need in this situation, since the tension in the string mixes with the radial component of gravity to generate the radial acceleration. We shall now go into the world of approximations and suppose that the oscillations' amplitude is modest. The issue cannot be addressed in closed form without this approximation. We may use $\sin \theta \approx \theta$ in Eq. (4.3), assuming θ is small. To acquire where the initial circumstances define. As a result, the pendulum moves in a simple harmonic manner at a frequency of $\omega = \sqrt{g/l}$. Therefore, the period is $T = 2\pi \sqrt{l/g}$. For sufficiently tiny values, the real motion is arbitrarily close to this amplitudes. The higher-order modifications to the motion in are based on the in the event that the amplitude is not minimal [5], [6].

Throughout your study of physics, you will encounter several instances when you will labor through a computation only to arrive to a straightforward equation of the type $\ddot{z} + 2\gamma\dot{z} + \omega_0^2 z = 0$, where 2γ is a positive amount that relies on different problem-related factors. When you come across an equation like this, you should rejoice because you can easily write out the solution: the answer for z must have the form $z = e^{\lambda t}$. The system experiences simple harmonic motion with a frequency equal to the square root of the coefficient of z , regardless of how difficult the system initially seems to be. If you end up with an equation that looks like $\ddot{z} + 2\gamma\dot{z} = 0$, then you know this whatever that coefficient may be. If you get after adding the courgette, then generally speaking, as long as the courgette is positive and has the dimensions of squared inverse time).

Dampened harmonic motion

Consider a spring with a spring constant of k and a mass m connected to the end of it. Let $F_f = -b\dot{x}$ (the subscript f here stands for "friction"; we'll reserve the letter d for "driving" in the next section) be the drag force applied to the mass in relation to its velocity. 4.3. Why do we investigate this dampening force, $F_f = -b\dot{x}$? First, since it is linear in $\dot{x}$, we can use it to solve for the motion. Additionally, it is a fully realistic force; typically, a drag force proportionate to velocity is experienced by an item travelling slowly through a fluid. Notably, the force represented by $F_f = -b\dot{x}$ is not the same as the force a mass would experience if it were put on a table with friction. The drag force would thus be basically constant.

Over-damping

Because there are two negative exponents in the expression, the motion for big t is zero. This better be the case since a true spring won't allow the motion to continue indefinitely. We would be aware that we were off if we had somehow managed to acquire a positive exponent.

The two components in are almost equal, and e^{-t} indicates that we practically get exponential decay if γ is just a little bit greater than ω . Since the first part, which has the less negative exponent, dominates in the case of (high damping), we effectively experience exponential decay. By roughly estimating as, we may be fairly quantitative about this. As a result, the exponential behaviour is as follows: $e^{-\gamma t/2}$. Due to, this decay is gradual (slow in comparison to $t^{-1/2}$), which makes sense if the damping is substantial. The mass slowly returns to its starting point, much like a weak spring submerged in molasses. ♣

Critical damping

We must use caution while resolving our differential equation in this unique situation. The answer is invalid because the process that results in Eq. As a result when the roots 1 and 2 are equivalent, there is only one viable solution, e^{-t} . Invoking a result from differential equation theory, we may argue that in this particular instance, the alternative solution has the form $t e^{-t}$. The movement. You can demonstrate that the amplitude does indeed drop with $C e^{-t} \cos(\tan^{-1}(\gamma/\omega))$. According to Castro (1986), this is the term for the curve that cuts across the extremes. However, this is effectively identical to $C e^{-t}$ for modest damping ($\gamma < \omega$). It is proportional to e^{-t} in any case [7], [8].

Critical damping, which occurs when $\gamma = \omega$, is the situation where the motion converges to zero in the shortest possible manner if we are given a spring with a fixed ω and examine the system for various values of γ . This is so because, in the underdamped situation ($\gamma < \omega$), the oscillatory motion's envelope moves like e^{-t} and approaches zero more slowly than e^{-t} does. Additionally, the term $t e^{-t}$ is the dominating element in the over damped situation. And as you can see, if $\gamma > \omega$, then $\gamma^2 > \omega^2$, meaning that this motion similarly approaches zero more slowly than e^{-t} . In many practical systems, such as screen doors and shock absorbers, where the objective is to have the system head to zero (without overshooting and bouncing about) as quickly as possible, critical damping is crucial.

Driven (and dampened) harmonic motion

We first need to learn how to solve a new kind of differential equation before we can look into driven harmonic motion. How can we resolve a problem of this nature? Due to the term on the right-hand side, this differential equation is an inhomogeneous one. It's not really physically demanding since the right side is complicated, but let's put it to the side for the time being. These kinds of equations often arise, but thankfully there is a simple—if sometimes messy—method for solving them. The process entails creating a fair assumption, plugging it in, and then observing the result to determine the condition. Since the $e^{i\omega t}$ is positioned on the right side. Let's assume that the answer to has the form $A e^{i\omega t}$. We will show that, among other things, A depends on ω . Introducing this hunch after eliminating the nonzero component.

A couldn't be solved since it was decided by the beginning circumstances. However, the objective of the current method is to solve for A in terms of the provided constants, with the ω in being a determined amount. In the equation's solution, Eq. No free constants can be established from the beginning circumstances. We've only got one option left, and we're stuck with it. For equation, the phrase "particular solution" is used without the ability to modify the equation's answer. How can we fulfil a random set of beginning conditions. Thankfully, Eq. The most generic response to Eq. does not correspond to the sum of our specific answer to equation is the most universal solution as well as the "homogeneous" answer we discovered in the answer is this sum, hence it is unquestionably a solution was specifically designed to

produce zero when connected to the left side. As a result, adding it to our specific solution has no impact on the equality of equation given that the left side is linear.

Let's put the origin of these equations aside for the time being. We'll perform a practical example later in this section, but for now, let's simply attempt to solve them. Although it is not required, we will assume that $2 > 0$ in this case. We'll also make the assumption that there aren't any damping or driving forces, even if several of the problems and exercises in this chapter do. The reason why the aforementioned equations are referred to be "coupled" is because both of them include x_s and y_s , and it is not immediately clear how to separate them in order to solve for x and y . These equations can be solved using at least two different approaches.

First approach: Finding certain linear combinations of the supplied equations for which good things occur may sometimes be simple, as in this instance. Only the combination of x and y , or their total, $x + y$, is involved in this equation is merely our trusty buddy, $z + 2z = 0$, when $z = x + y$. We may also use resulting in the equation where the A_i 's and B_i 's are split evenly. This solution's approach was to just play about and attempt to create differential equations using only one set of the variables. This gave us the opportunity to record the well-known answer to these combinations.

We were able to figure out the equations for x and y . It turns out, however, that creating the most intriguing thing we've done. The normal coordinates of the system are the combination. These pairings vibrate at a single, pure frequency. In general, the motion of x and y will seem complex, and it may be challenging to discern that the motion is really composed of just the two frequencies.

However, even if x and y behave in disagreeably, if you plot the values over time for any motion of the system, you will discover lovely sinusoidal graphs. Second method: It was pretty simple to predict which combinations would result in equations involving only one pair of x and y . But there must be physics issues where guesswork is more difficult. So what do we do? Fortunately, there is a foolproof approach to finding x and y . It goes on like this.

The existence of solutions for x and y that have the same t dependency is not immediately apparent, but let's give it a go and see what happens. Although i has been explicitly added to the exponent, there is no loss of generality. If i is fictitious, the exponent will be true. Whether or not you insert i depends on your choice. Only if the matrix is not invertible does this homogeneous equation for A and B have a nontrivial solution, meaning one where A and B are not both 0. If it were invertible, we could multiply through by the inverse to get $(A, B) = (0, 0)$, which proves that it is not invertible. A matrix becomes invertible when? There is a simple (but time-consuming) technique for determining the inverse. Taking cofactors, transposing, and dividing by the determinant are the steps involved. The division by the determinant step is the one that concerns us in this case since it means that the inverse exists only if and when the determinant is not zero. As a result, we can observe that only has a nontrivial solution if the determinant is zero. Since we are looking for a nontrivial solution, we must have

The roots of this quadratic equation in the number two are equal to and equal to two. Thus, we have identified four different sorts of solutions. If $=$, we can re-insert this into to get $A = B$. The outcome is the same for both equations. The purpose of putting the determinant equal to zero was simply to achieve this.) Additionally, if $= 2$, then results in $A = B$. (Once again, the equations are unnecessary.) Keep in mind that we can only get A/B 's ratio by solving, not

for A or B individually. According to the concept of superposition, we add together our four answers and find that x and y assume the general form (expressed in vector form for ease of accounting and simplicity).

Top speed

At point x_0 , a mass at the end of a spring (with a natural frequency ω_0) is propelled away from rest. The experiment is repeated, but this time the apparatus is submerged in a fluid with a damping coefficient that causes the motion to be excessively dampened. Calculate the ratio of the maximum speed in the first scenario to the second. What is the ratio at the strongest damping limit ($\gamma = 2\omega_0$)? In the critical damping limit ($\gamma = \omega_0$)?

Exponential force

A mass m particle experiences a force $F = F_0 e^{-\gamma x}$. Position and beginning speed are both at zero.

Unequal masses

Two masses, m and $2m$, and three identical springs are positioned between two walls. Discover the default modes. Weakly linked as indicated in Fig., three springs and two equal masses are positioned between two walls. The two external springs' spring constants, k , are substantially less big than the middle spring's constant. Let x_1 and x_2 represent the left and right masses' locations in relation to their equilibrium positions, respectively. Show that x_1 and x_2 may be expressed as (assuming k) if the starting locations and if both masses are released from rest.

Driven mass on a circle

The movement of two similar masses m is restricted to a horizontal hoop. The masses are connected by two identical springs with spring constant k that round the hoop. $F \cos \omega t$ is a driving force acting on one mass. Find the specific answer to the mass motion problem.

Springs on a circle

(A) A horizontal hoop is used to limit the motion of two similar masses, m . The masses are connected by two identical springs with spring constant k that round the hoop. Discover the common modes.

(b) The motion of three similar masses is restricted by a hoop. Three similar springs, join the masses and round the hoop. Discover the common modes. Using N identical masses and N identical springs, do the general case [9], [10].

Angled rails

Two horizontal frictionless rails with a 2θ angle between them are required to transport two particles of mass m along them. A spring with a spring constant of k , whose relaxed length is l at the location connects them. What is the oscillation frequency for the motion if the spring stays parallel to the shown position? Each of two springs has an equilibrium length of l and a spring constant of k . As shown they are both extended a distance d and joined to a mass m and two walls. The right spring constant is suddenly and miraculously adjusted to $3k$, but the relaxed length stays at l . What is the final value assume that x is at its starting value. The springs are at their equilibrium length. The mass oscillates with amplitude d along the line of the springs. The right spring is taken out at the instant when the mass is at position $x = d/2$. Rails or tracks that are not precisely horizontal but are inclined at an angle with respect to the

horizontal plane are referred to as angled rails. These inclined rails are often used in a variety of settings, including engineering, sports, and transportation. Several applications for angled rails are shown here:

Rail Transportation:

Tracks sometimes need to be placed on slopes or inclines when it comes to railroads, particularly in mountainous or hilly areas. Trains can go across steep terrain thanks to the inclined rails, and an incline or drop makes it easier to effectively handle elevation changes

Roller coasters:

In order to give thrilling and exciting rides, roller coasters are constructed with inclined rails. By adjusting the rails' angle, thrilling twists and turns are made possible, including the steep drops and abrupt curves. Angled rails are used to build slopes and terrain parks for skiing and snowboarding. These slanted rails are used by skiers and snowboarders for both recreational and competitive reasons.

Manufacturing and Conveyor Systems:

Production lines in manufacturing often transfer materials or products up or down multiple levels inside a facility using tilted conveyor belts or rails. This makes it possible for warehouses and factories to handle materials effectively. In physics experiments and instructional displays, inclined planes or rails are used to examine how gravity affects things. These set-ups are helpful for introducing ideas relating to friction, acceleration, and velocity.

Agriculture:

To create level areas on hills, terraced farming sometimes uses inclined rails or steps. In hilly areas, this helps to maximise land utilisation and avoid soil erosion.

Extreme Sports:

Angled rails and ramps are utilised in extreme sports like skateboarding to accomplish tricks and stunts. These incline surfaces are used by skaters for jumps, grinds, and other tricks.

Aerospace:

Angled launch rails are used in aerospace engineering to launch aircraft from aircraft carriers. These rails aid in supplying the required takeoff acceleration. These rails' angles are specifically developed and constructed to meet the demands of each application, assuring their safety, effectiveness, and use. Angled rails enable a variety of modes of transportation, entertainment, and industrial activities. They are essential in many different sectors [11], [12].

CONCLUSION

REFERENCES:

- [1] H. S. Lee *et al.*, "Astrocytes contribute to gamma oscillations and recognition memory," *Proc. Natl. Acad. Sci. U. S. A.*, 2014.
- [2] M. Lemieux, J. Y. Chen, P. Lonjers, M. Bazhenov, and I. Timofeev, "The impact of cortical deafferentation on the neocortical slow oscillation," *J. Neurosci.*, 2014.

- [3] Y. Yanovsky, M. Ciatipis, A. Draguhn, A. B. L. Tort, and J. Brankač, “Slow oscillations in the mouse hippocampus entrained by nasal respiration,” *J. Neurosci.*, 2014.
- [4] J. Jacobs, “Hippocampal theta oscillations are slower in humans than in rodents: Implications for models of spatial navigation and memory,” *Philosophical Transactions of the Royal Society B: Biological Sciences*. 2014.
- [5] C. M. A. Chen *et al.*, “GABA level, gamma oscillation, and working memory performance in schizophrenia,” *NeuroImage Clin.*, 2014.
- [6] V. V. Moca, D. Nikolić, W. Singer, and R. C. Mureşan, “Membrane resonance enables stable and robust gamma oscillations,” *Cereb. Cortex*, 2014.
- [7] D. Dvorak and A. A. Fenton, “Toward a proper estimation of phase-amplitude coupling in neural oscillations,” *J. Neurosci. Methods*, 2014.
- [8] H. E. Rossiter, E. M. Davis, E. V. Clark, M. H. Boudrias, and N. S. Ward, “Beta oscillations reflect changes in motor cortex inhibition in healthy ageing,” *Neuroimage*, 2014.
- [9] C. von Nicolai, G. Engler, A. Sharott, A. K. Engel, C. K. Moll, and M. Siegel, “Corticostriatal coordination through coherent phase-amplitude coupling,” *J. Neurosci.*, 2014.
- [10] C. J. Watras *et al.*, “Decadal oscillation of lakes and aquifers in the upper Great Lakes region of North America: Hydroclimatic implications,” *Geophys. Res. Lett.*, 2014.
- [11] R. J. Gardner, F. Kersanté, M. W. Jones, and U. Bartsch, “Neural oscillations during non-rapid eye movement sleep as biomarkers of circuit dysfunction in schizophrenia,” *Eur. J. Neurosci.*, 2014.
- [12] A. M. Bastos, F. Briggs, H. J. Alitto, G. R. Mangun, and W. Martin Usrey, “Simultaneous recordings from the primary visual cortex and lateral geniculate nucleus reveal rhythmic interactions and a cortical source for gamma-band oscillations,” *J. Neurosci.*, 2014.

CHAPTER 5

CONSERVATION OF ENERGY AND MOMENTUM

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ABSTRACT:

Conservation laws are very significant they are very valuable in determining what is happening in a physical system, both statistically and qualitatively. When we use the word "conserved," we refer to anything that has remained the same across time. The possible ultimate movements are significantly constrained if a specific amount is preserved, as it is when a ball rolls around in a valley or when a set of particles interacts. We can solve the issue if we can limit the ultimate movements to just one possibility by writing down enough conserved quantities, which is usually possible, at least for the systems we'll be interested in. Two of the fundamental conservation rules in physics are the conservation of energy and momentum go over a third, conservation of angular momentum. It should be emphasised that using the principles of energy and momentum conservation is not always essential. These conservation rules will be derived from Newton's laws. Therefore, if you were to start from scratch, you could always (in theory) use $F = ma$, etc. At best, you would quickly become tired of this strategy. And in the worst case scenario, you would give up after concluding that the issue was unsolvable. For instance, attempting to analyse the collision between two shopping carts—whose contents are free to move around—by examining the forces on all the different things would be fruitless

KEYWORDS:

Conservative, Energy, Force, Potential, Work

INTRODUCTION

However, you may learn a lot rapidly thanks to the conservation of momentum. The benefit of conservation rules is that they greatly simplify computations and provide you a way to gauge the system's general qualitative behavior [1], [2].

Energy conservation in a single dimension

Consider a force that solely relies on location and exists in one dimension for the time being. As a result, $F = F(x)$. If we express an as $v dv/dx$ and the force operates on a particle with mass m , then $F = ma$. From a specified position x_0 where the velocity is v_0 to an arbitrary point x where the velocity is v , we may separate the variables here and integrate. The outcome is in where $E = mv^2/2$. E relies on v_0 , thus choosing x_0 has an impact on v_0 as well since various x_0 s often result in different v_0 s. Following the steps in Section 3.3 for a function that relies just on x is all that has been done in this case. If the potential energy, $V(x)$, is now defined as

Here, we define the kinetic energy as the first term. The particle's total kinetic and potential energy is constant since this equation is true across the whole range of the particle's motion. Therefore, the whole amount of energy is preserved. A particle's speed changes depending on whether it obtains or loses potential energy. Common examples of potential energy for the gravitational force (mg), with y_0 set to zero, and $kx^2/2$ for a Hooke's law spring force (kx), with x_0 set to zero.

Oh, this is just fantastic, Jack thought as he stumbled over some debris in the beach. He altered his capacity. torrential, kinetic, before grasping Jill's hand, however. That is what really occurred on that hill, then. Of course, people don't suddenly "tumble after" you out of nowhere. The arbitrary selection of x_0 has an impact on both E and $V(x)$ for a particle moving in a certain way. This suggests that E and $V(x)$ are meaningless on their own. The only significant quantity is the difference between E and $V(x)$. It is equal to the kinetic energy); this variation is unaffected by the selection of x_0 . However, we must choose an arbitrary x_0 in order to be concrete in a particular arrangement, thus we must remember to mention the x_0 we've selected. For instance, it is illogical to state that an object's gravitational potential energy

Here, it is evident that only variations in potential energies are significant. The work-energy theorem is created if we define the integral in this equation as the work done on the particle as it goes from x_1 to x_2 . The work done on a particle between x_1 and x_2 is equal to the change in the particle's kinetic energy between those two positions. If the force and motion are both directed in the same direction (that is, if $F(x)$ and dx . The work is positive and the speed rises have the same sign. The work is negative and the speed drops if the force is applied in the direction perpendicular to the motion [3], [4].

Therefore, anytime anybody speaks about gravitational potential energy in a setup on the surface of the earth, it is usually recognized that the ground is the reference point as it becomes tedious to constantly repeating "with respect to the ground". For convenience's sake, as we shall see in the first example below, $r =$ is taken to be the reference point if, on the other hand, the experiment extends to distances far from the earth.

This theorem only applies to point particles without internal structure in the form given here. For the broad theorem, see the section below under "Work vs. Potential Energy." thinking about Eq. Let's create after presuming that we have picked the reference point x_0 for the potential energy (and possibly added a constant to $V(x)$ just for fun). The constant E line and the $V(x)$ curve (which we may find if we are provided, for example, the beginning location and speed). The kinetic energy is then determined by the difference between E and $V(x)$. The locations where $V(x) > E$ are where the particle cannot go. The "turning points" are the locations where $V(x) = E$, when the particle pauses and reverses direction. In the illustration, the particle oscillates back and forth while being stuck between x_1 and x_2 . In this sense, the potential $V(x)$ is very helpful since it clarifies the fundamental characteristics of the motion.

Since only changes in the potential—which are unrelated to x_0 —have any significance, it may seem stupid to propose a particular x_0 as a point of comparison. It's similar to taking the difference between 17 and 8, which is equal to determining their sizes in relation to 5, which are 12 and 3, and then subtracting 3 from 12 to get 9. However, as integrals are more difficult to do than straightforward subtraction, it is preferable to perform the integral once and for all, labelling each point with a specific number $V(x)$, and then take differences between the V 's as necessary.

It is simple to take the derivative of $V(x)$ to get $F(x)$. However, it could be challenging (or impossible) to carry out the integration in Eq. given $F(x)$. Write $V(x)$ in closed form in (5.3). However, this is not a major worry. Given that the force is merely a function of x , the function $V(x)$ is clearly defined and may, if necessary, be numerically calculated with any level of precision [5], [6].

DISCUSSION

Assume that M and m are two point masses that are spaced apart by r . According to Newton's law of gravitation, there is an attractive force between them with a magnitude of GMm/r^2 . Therefore, the system's potential energy at separation r , calculated in relation to separation r_0 , is where the attractiveness of the force is what causes the negative sign in the integrand. The second term vanishes when r_0 is chosen as the value because of this. From this point forward, it is assumed that this $r_0 =$ reference point has been selected. Consequently what is the gravitational potential energy of a mass m at height y , with relation to the ground, as an example (gravity near the earth)? We are aware that it is difficult, but let's try it that way anyhow holds if M is the earth's mass and R is its radius.

And the second line was obtained by using the Taylor series approximation. We have also taken use of the fact that, in terms of gravity, a sphere may be thought of as a point mass. This is what demonstrates. We can observe that the potential energy discrepancy. Of fact, this has just been a circular discussion. Imagine a ball rolling around in a valley or on a hill to help you visualize a possible $V(x)$. For instance, the potential of a common spring is $V(x) = kx^2/2$ (producing the Hooke's-law force, $F(x) = dV/dx = kx$), and if we visualize a valley whose height is provided by $y = x^2/2$, we may have a good understanding of what is happening. Thus, $mgy = mgx^2/2$ is the gravitational potential of the ball. The required potential is obtained by selecting $mg = k$. The configuration seems to be similar to the original spring if we then examine the ball's motion as projected along the x axis.

The two configurations are different, despite the fact that this similarity aids in the visualization of the motion's fundamental characteristics. The specifics of this fact are reserved but the observation that follows should persuade you that they are in fact distinct. In both configurations, allow the ball to be released from rest at a high value of x . The spring's force, kx , is therefore enormous. However, only a small portion of mg , or $\cos$, acts in the x direction on the particle in the valley, where is the angle of the valley at that location. For tiny oscillations around the valley's bottom, the setups are roughly the same [7], [8].

Republican troops

The work a force exerts on a particle is denoted as $\int W F dx$ for any given force (which may depend on x , v , t , and/or other variables). We can determine the total work done on the particle by all the forces if the particle begins at x_1 and ends up at x_2 , regardless of how it gets there (it could speed up or slow down, or reverse direction a few times). Some forces don't care how a particle travels since they nevertheless perform work. Due to the integral, a force that relies solely on location (in one dimension) has this characteristic. Only the endpoints are dependent. The particle's path from x_1 to x_2 has no bearing on the $W = \int F dx$ integral, which is the (signed) area under the F vs. x graph.

For other forces, the amount of work is determined by the particle's motion. The same is true for forces dependent on t or v , since it concerns when or how rapidly the particle transitions from x_1 to x_2 . Friction is a frequent illustration of such a force. The frictional effort required to move a brick from x_1 to x_2 on a table is equal to $mg|x|$. However, if you spend an hour wriggling the block back and forth before you eventually reach x_2 , then friction is doing a lot of negative work. There is never any cancellation since friction constantly opposes the motion and always has a negative contribution to the $W = \int F dx$ integral. The outcome is a significant negative number.

The problem with friction is that, despite the fact that the mg force (which is a subset of position-only dependent forces) seems to be a constant force, it really isn't. Depending on which direction the particle is travelling at a certain place, the friction may point to the right or to the left. Therefore, friction depends on velocity. It is true that it depends only on the velocity's direction, but that is sufficient to destroy the position-only dependency. The work performed on a particle between two locations is now defined as a conservative force when it does not matter how the particle travels. The point we're making here is that even though we can calculate the work done by any force, it makes sense to talk about the potential energy associated with a force. This is because, as we learned from the discussion that came before it, a one-dimensional force is conservative if and only if it depends only on x (or is constant).

Only in cases when the force is moderate. This is true because we want to be able to assign a distinct number, $V(x)$, determined by $V(x) = \int_{x_0}^x F dx$, to each value of x . This integral wouldn't be constant if it depended on how the particle moves from x_0 to x . Clearly specified, therefore we couldn't determine a value to use in place of $V(x)$. As a result, we only discuss potential energies when they are linked to conservative factors. Talking about the potential energy connected to a friction force, in instance, is absurd. The fact that the gravitational potential energy, or mgz , is independent of the direction a particle follows, even in two or three dimensions, is an important one. As a result, even if the particle goes in a convoluted route, just. In contrast, a conservative force in two or three dimensions must also meet another condition in addition to being position-dependent [9], [10].

The vertical z component of the displacement is important for figuring out how much work gravity is doing. If we divide the route into many little parts, we can sum up all of the tiny $mg(dz)$ terms to get the overall amount of work done by gravity. However, regardless of the route, the total z is always equal to the sum of all the dz s. Therefore, the change in gravitational potential energy is always only mgz , regardless of what the particle does in the two horizontal directions. Therefore, in three dimensions, the gravitational force is a moderate force. That this is a specific example of a more general finding.

We need another equation since there are two unknowns in this situation: v and θ . This will be the (basically) final circular motion's radial $F = ma$ equation. The motion may always be represented by a horizontal circle that steadily declines over time since the pole is so thin. The total force in the vertical direction is 0 since there is basically no motion in this direction. As a result, mg is basically the vertical component of the tension. The ultimate circular motion is described by the $F = ma$ equation, where the horizontal component is then $mg \sin \theta$ the result of equating our two formulas. It's interesting that this angle doesn't rely

Effort versus available energy

If a ball is dropped, does it move faster because the gravitational force is acting on it or because the gravitational potential energy of the ball is dwindling? Both, or more specifically, either. For conservative forces at least, work and potential energy are two distinct methods of describing the same thing. Either approach to reasoning produces the right outcome. Be cautious not to "double count" the impact of gravity on the ball by using both justifications. Just like with $F = ma$ and free-body diagrams, it is vital to name your system when dealing with work and energy, as we'll see in the example below. Which nomenclature you use depends on what you designate your "system."

Work-energy theorem applies to a single particle. What if we are discussing the work that has been done on a system that is made up of several parts? According to the general work-

energy theorem, the work that external forces exert on a system equals the change in energy of the system. Heat comes into this category since it is just the random motion of molecules, although this energy may also take the form of overall kinetic energy, internal potential energy, internal kinetic energy. Therefore, the generic work-energy theorem may be expressed as according to theorem, a point particle has no internal structure, thus we only have the first of the three terms on the right-hand side. However, have a look at the example below to understand what occurs when a system has internal structure.

The lesson here is that depending on the system you use, you may see a configuration in a variety of ways. Work in one context may manifest as potential energy in another. In actuality, working in terms of potential energy is often more practical. Therefore, for a dropped ball, people often think of gravity as an internal force inside the earth-ball system rather than an external force acting on the ball system (whether consciously or unconsciously). Generally speaking, "conservation of energy," which is often utilised in settings involving gravity and/or springs, is an easy theory to put into practise (and you'll have plenty of opportunity to do so in the problems and exercises for this chapter). It turns out that you can typically disregard all of these concerns regarding your job and choosing a system.

To be sure we're on the same page, let's take a look at one more illustration. Take into account a vehicle that is braking but not sliding. The automobile slows down as a result of the ground's friction force with the tyres. However, since the earth isn't moving, this force has no effect on the automobile and operates across no distance. As a result, Eq. (5.13), on its left side, has no external work. Therefore, the right side is similarly zero. In other words, the car's overall energy remains constant. This is accurate because, despite the car's overall kinetic energy declining, the brake discs and pads' internal kinetic energy rising in the form of heat. In other words, the total energy stays constant and $K = K_{\text{internal}}$. The negative aspect of this procedure is that the energy used to create heat is wasted and cannot be recovered to increase the car's total kinetic energy. Much more logically, the whole kinetic energy should be transformed into some kind of internal potential energy (i.e., $K = U$), which may then be transformed back into the total kinetic energy. In hybrid vehicles, the whole kinetic energy is converted into chemical potential energy in a battery.

Three-Dimensional conservative forces

There is one issue that occurs in 3-D that we weren't concerned about in 1-D for a force that solely relies on location (as we have been assuming). There is just one path in 1-D that connects x_0 and x . The motion itself could entail accelerating, decelerating, or turning around, but the route is never allowed to deviate from the line between x_0 and x . However, there are an endless number of paths from r_0 to r in 3-D.

The potential $V(r)$ must be well specified in order to have any significance or use. It must be path-independent, in other words. We refer to the force connected to such a potential as a conservative force, just as in the 1-D situation.

Now let's look at the conservative 3-D forces. Conservative forces in three dimensions are those in physics that can be characterised by a scalar potential energy function and adhere to the laws of mechanical energy conservation.

These forces operate on things in three dimensions, and their key feature is that they function regardless of the route they follow to move an item from one place to another. Here is a more thorough justification:

Scalar Potential Energy Function:

In three-dimensional conservative forces, a scalar function that is specified at every point in space (x , y , and z) is known as $V(x, y, z)$. The force connected to this function, which is known as the potential energy, may be calculated as the potential energy function's negative gradient. In this case, the force is given as a vector by the gradient operator, which acts on the potential energy function.

Work Independence of route:

One of the fundamental characteristics of conservative forces is that the amount of work that the force does when an item travels between two places A and B remains constant, independent of the precise route that is travelled. The work-energy theorem describes this feature, which is a direct result of the force's conservatism where E is the change in mechanical energy of the item (kinetic energy plus potential energy) between those two positions and W_{AB} is the work performed by the conservative force from point A to point B.

Three-Dimensional Conservative Forces Examples

The gravitational force is an excellent illustration of a three-dimensional conservative force. A potential energy function (gravitational potential) that is dependent on an object's location in three dimensions may be used to explain it. Another example is the electrostatic force that exists between charged particles. With the help of the electric potential energy function, it may be explained. The force a spring exerts in three dimensions is conservative, and its potential energy relies on the displacement in each of the three spatial dimensions.

Conservation of Mechanical Energy:

As long as no non-conservative forces (such as friction) are present, the total mechanical energy (kinetic energy plus potential energy) of an isolated system stays constant throughout time. This conservation of energy is an effective tool for examining how things move in different physical systems three-dimensional conservative forces are essential to physics because they enable mechanical energy conservation and make it easier to analyse complicated systems. They have the feature of work independent of path and may be characterized by scalar potential energy functions.

The total mechanical energy of an isolated system stays constant if only conservative forces are acting on it, according to the conservation of mechanical energy, a basic tenet of physics. This principle is a result of the conservation of energy, which is a basic rule of nature, and it is derived from the work-energy theorem. The two parts that make up mechanical energy are as follows:

Kinetic Energy (KE):

Kinetic energy is the force that propels an item through space. The formula, which is dependent on the object's mass (m) and velocity (v), is as follows:

$$KE = 0.5 * m * v^2$$

Potential Energy (PE) is the energy connected to an object's placement or arrangement inside a force field. The gravitational potential energy and elastic potential energy are the two most prevalent types of potential energy. The formula for gravitational potential energy, which is

determined by the height (h) and gravitational force (g), respectively, of an object, is as follows: PE gravitational is equal to $m \cdot g \cdot h$ [11], [12].

CONCLUSION

Elastic potential energy is determined by how much a spring or other elastic material is deformed and is represented as: where x deviates from the equilibrium position and k is the spring constant. The principle of mechanical energy conservation asserts that the total of an object's kinetic and potential energy stays constant in the absence of non-conservative forces (such as friction or air resistance) KE plus PE equals total mechanical energy (E). This indicates that an object's mechanical energy is conserved when it travels inside a conservative force field, such as when it is affected by gravity or a spring system. Although the kinetic and potential energies of the item may fluctuate as its location changes, their combined total, or mechanical energy, stays constant. In physics and engineering, the conservation of mechanical energy is a useful tool for understanding the behavior of systems like pendulums, roller coasters, and planetary motion. While upholding the idea of energy conservation, it enables us to forecast the motion of objects and comprehend how energy is moved between kinetic and potential forms inside a system.

REFERENCES:

- [1] A. Narayanaswamy and Y. Zheng, "A Green's function formalism of energy and momentum transfer in fluctuational electrodynamics," *J. Quant. Spectrosc. Radiat. Transf.*, 2014.
- [2] K. Zubovas and S. Nayakshin, "Energy- and momentum-conserving AGN feedback outflows," *Mon. Not. R. Astron. Soc.*, 2014.
- [3] N. J. Popławski, "The energy and momentum of the Universe," *Class. Quantum Gravity*, 2014.
- [4] C. M. Dodson, J. A. Kurvits, D. Li, and R. Zia, "Wide-angle energy-momentum spectroscopy," *Opt. Lett.*, 2014.
- [5] P. Goebes *et al.*, "Momentum or kinetic energy - How do substrate properties influence the calculation of rainfall erosivity?," *J. Hydrol.*, 2014.
- [6] A. Herdegen, "On Energy-Momentum Transfer of Quantum Fields," *Lett. Math. Phys.*, 2014.
- [7] J. H. Jung, U. Yakhshiev, and H. C. Kim, "Energy-momentum tensor form factors of the nucleon within a π -p- ω soliton model," *J. Phys. G Nucl. Part. Phys.*, 2014.
- [8] S. Kawai and Y. Nakayama, "Improvement of energy-momentum tensor and non-Gaussianities in holographic cosmology," *J. High Energy Phys.*, 2014.
- [9] H. Nikolic, "The trivial solution of the gravitational energy-momentum tensor problem," *arXiv*, 2014.
- [10] P. Leitner, M. F. Heyn, I. B. Ivanov, S. V. Kasilov, and W. Kernbichler, "Effect of energy and momentum conservation on fluid resonances for resonant magnetic perturbations in a tokamak," *Phys. Plasmas*, 2014.
- [11] I. Zeljkovic *et al.*, "Mapping the unconventional orbital texture in topological crystalline insulators," *Nat. Phys.*, 2014.
- [12] T. Schmitt, F. M. F. De Groot, and J. E. Rubensson, "Prospects of high-resolution resonant x-ray inelastic scattering studies on solid materials, liquids and gases at diffraction-limited storage rings," *J. Synchrotron Radiat.*, 2014.

CHAPTER 6

UNDERSTANDING THE CONCEPT OF GRAVITY

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ABSTRACT:

Since ancient times, scientists and intellectuals have been captivated by gravity, the unseen force that controls astronomical body motion and defines the universe's structure. This abstract explores the key elements of gravity, including everything from its historical importance and discovery to its underlying principles and significant consequences for our knowledge of the universe. All things with mass are attracted to one another by gravity, a force of attraction that Sir Isaac Newton memorably described in his law of universal gravitation. This force not only keeps us firmly anchored on the surface of the Earth, but it also controls the dynamics of galaxies across the great expanse of the universe and the orbits of planets around the Sun. Albert Einstein's theory of general relativity, which defined gravity as the curvature of space time brought about by large objects, revolutionized our understanding of gravity in the field of contemporary physics. The prediction and subsequent confirmation of phenomena like gravitational lensing and gravitational waves, which have opened up new vistas for the study of the universe, are only two examples of the astonishing insights this theory has brought. Gravity also has a major impact on cosmology, affecting the universe's creation, development, and ultimate destiny. It influences the cosmic microwave background radiation, the afterglow of the Big Bang, and the distribution of matter in galaxies and clusters. It also contributes to the cosmos' expansion. The universality of gravity, the universe's deep impact on it, and its continued significance to both science and space travel are all highlighted in this abstract. The study of gravity continues to enthral the scientific community, providing many chances to solve the universe's riddles, from comprehending Earth's gravity to exploring the gravitational pulls of black holes and dark matter.

KEYWORDS:

Collision, Energy, Force Mass, Momentum

INTRODUCTION

What force results from the substitution of a sphere with a radius R and mass M for the point mass M ? The answer is that it is still GMm/r^2 (assuming that the sphere is spherically symmetric, which means that the density depends solely on r). As long as we take into account a mass m outside the sphere, a sphere behaves in terms of gravity exactly like a point mass at its centre. To put it mildly, this outcome is really pleasant. The cosmos would be far more intricate than it is if this were not the case. Particularly, it would be far more difficult to define how planets and other similar objects move [1], [2].

It turns out to be far simpler to calculate the potential energy owing to a sphere and then take the derivative to derive the force than to directly compute the force, thus this is the path we will pursue to show that spheres act like points in terms of gravity.⁹ Therefore, this is the approach we will use. Since a sphere is the sum of many such shells, it will do to show the outcome for a thin spherical shell. The method we'll use to determine the potential energy caused by a spherical shell at a point P involves cutting the shell into rings as illustrated in

Fig. 5.7. Let the radius of the shell be R , the position of P relative to the shell's centre be r , and the ring form the indicated angle. R , r , and θ are functions that determine the distance, d , between P and the ring. The location is as follows. In Fig. Segment AB 's length is $R \sin \theta$, whereas segment BP 's length is $r R \cos \theta$.

Naturally, these forces are directed in a radial fashion. If P is outside of a particular sphere, then the force there is GMm/r^2 , where M is the total mass of the sphere. A solid sphere is made up of several spherical shells. Even if the shells have various mass densities, this conclusion still remains true (albeit each shell must have a uniform density). Be aware that the gravitational attraction between two spheres is the same as if point masses were used in lieu of both. Two applications of our "point mass" finding lead to this conclusion [3], [4].

Newton examined both the numerical data and the empirical observations. But in course, we get the same force, he countered from a spherical object and a point mass!" Because all shells outside of this area yield zero if P is within a specific sphere, the only material that matters is the mass inside a concentric sphere via P force as determined by the second equation. For the sake of gravity, the substance "outside" of P does not exist. The absence of force inside a spherical shell is not immediately apparent. Look at Fig.'s point P . 5.9. Due to the $1/r^2$ dependency, a mass dm piece on the right side of the shell exerts a stronger force on P than one on the left. However, the figure shows that the left side is heavier than the right. As you can see, these two impacts perfectly cancel one another.

The Cavendish investigation

How can we calculate G 's numerical value we can calculate G if we can create a setup for which we are aware of the values of F , M , m , and r . The first idea that springs to mind is to make use of the fact that an item on the surface of the earth experiences a gravitational force that is known to be $F = mg$. This is combined with Eq. (5.37) results in $g = GME/R^2$. Since g and R , have known values, this identifies the nature of the GME product. Unfortunately, this knowledge is of little value to us since we lack the knowledge of the mass of the earth (without first knowing G , which we're presuming we don't now know; see the last paragraph in this section). For all we know, the earth's mass may be 10 times more than what we now estimate it to be, and G could be 10 times smaller [5], [6].

Utilizing a setup with two known masses is the only method to determine G . However, this then creates the issue because the generated force is really little. Finding a technique to quantify the minuscule force between two known masses is the last remaining job. By carrying out an incredibly delicate experiment (which was developed a few years earlier by John Michell, but he passed away before being able to carry it out), Henry Cavendish was able to overcome this difficulty in 1798. A extremely thin wire is suspended from its ends by two masses M . The dumbbell begins with no twist in the wire, and two extra masses M are subsequently added (fixed) in the places indicated. The dumbbell is free to twist, but if it does, the wire will produce a little restoring torque. The masses of the dumbbell are attracted to these masses, which causes the dumbbell to rotate anticlockwise. Back and forth oscillations of the dumbbell will occur until it ultimately settles at a little angle relative to its starting position.

The torque generated by the twist in the wire acts on the dumbbell in the form of $\tau = b\theta$ (with anticlockwise torque assumed to be positive), where b is a constant that depends on the thickness and composition of the wire. The $F = kx$ Hooke's-law result and this linear relationship between and hold for small for the same reasons. The gravitational force between

each pair of masses is represented by the formula GMm/d , where d is the distance between the centres of each pair of masses. As a result, the torque created by the two gravitational forces acting on the dumbbell is $2(GMm/d^2)l$, where l is one-half the dumbbell's length. Demanding that there be no total torque applied to the dumbbell. Since the time of Eratosthenes, or around 250 BC, the radius of the earth has been known (at least roughly). See Rawlins (1979) for an intriguing method to measure it yourself.

Rather than measuring G , Michell and Cavendish really meant for the experiment to measure the density of the earth; for further information, see Clotfelter (1987). However, as we'll see in a moment, this is comparable to measuring G . You may wish to read this part again after Chapter 8 as we won't discuss torque until then. Even if you are unfamiliar with rotations, the overall arrangement should be obvious. We'll use some findings related to rotational dynamics here.

If we can find b , we're done since we know all the other parameters on the right-hand side except for that one. Given how little the torque in the wire is, it is exceedingly challenging to measure b directly with any kind of precision. Fortunately, there is a cunning method to figure out b that requires using an oscillation from our plan. As the rotational equivalent of Newton's second law, $F = ma$, the fundamental equation for rotations is $\tau = I\alpha$, where I is the moment of inertia (which we can compute for the dumbbell). Therefore, if the torque has the form $\tau = b\theta$ for small θ , $\tau = I\alpha$ becomes $b = I\omega^2$ [7], [8].

DISCUSSION

The frequency of the oscillations is $\omega = \sqrt{b/I}$ since this is a classic basic harmonic oscillator equation. Since the oscillations' period, $T = 2\pi/\omega$, can be measured while the dumbbell is settling, all that is required to compute b is for $b = I\omega^2$ to equal $I(2\pi/T)^2$. (The time T is enormous because b is small on the scale of things; otherwise, there wouldn't be any discernible twist in the wire.) Because we know G (as well as g and R), we can apply $g = GME/R^2$ to compute the mass of the earth, ME , which is why the Cavendish experiment is also known as the "weighing the earth" (or maybe the "massing the earth") experiment. Without inspecting every cubic metre of the earth's interior, which is obviously impossible, the only way to determine ME is to first determine G , as we have done.¹³ It's interesting to note that the resulting value of ME , which is roughly 6×10^{24} kg, results in an average density of the earth of about 5.5 g/cm^3 . This means that there must be something exceedingly dense deep within the earth as it is greater than the density of the earth's crust and mantle. Thus, the Cavendish experiment, in which masses are suspended on a wire, miraculously reveals information about the earth's core [9], [10].

Momentum conservation

According to Newton's third law, there is an equal and opposite force for every force. Alternatively stated, if F_{ba} is the force that particle b experiences as a result of particle a , and F_{ab} is the force that particle a experiences as a result of particle b , then $F_{ba} = -F_{ab}$ at all times. This law has significant effects on momentum, when $p = mv$. Think about the interactions between two particles over time. Assume they are unaffected by external factors. By integrating Newton's second law, $F = dp/dt$, we can observe that the overall change in a particle's momentum equals the passage of time. Refer to Celnikier (1983) if you wish to find ME without utilising g or R (but still requiring G , of course, as ME only arises via the combination GME). See Brush (1980) for a thorough analysis of the earth's core the force

operating on it is vital to it. The impulse is the name of this integral. Now, if we apply the third rule, $F_{ba} = F_{ab}$, we discover

This is the claim that the overall momentum of this two-particle system operates independently of time and is preserved. Observe that equation since 5.50 is a vector equation, it really consists of three equations: p_x , p_y , and p_z conservation. The third law of Newton makes a claim about forces. But $F = dp/dt$ connects force and momentum. Thus, the conservation of momentum is effectively presupposed by the third law aforementioned "proof" is barely a proof. You could question whether momentum conservation is something you can prove or if you have to assume it, which is essentially what we have done since we have just accepted the third law.

It's difficult to define exactly what separates a postulate from a theorem. A postulate may be a theorem for one individual and vice versa. Your presumptions must begin somewhere. The third law is where we decided to start. The starting point is different in the Lagrangian formalism, and momentum conservation is inferred as a result of translational invariance (as we will see). In that framework, it so seems to be more like a theorem. However, one thing is for sure. It is impossible to establish momentum conservation for two particles using random forces since it is not a given. The total momentum of the two particles may not be preserved, for instance, if two charged particles interact in a certain manner via the magnetic fields they generate. Where is the lost energy? In the electromagnetic field, it is removed. Although the system's overall momentum is preserved, the fact that the system comprises of the two particles and an electromagnetic field is what matters most. In other words, it is the electromagnetic field—not the other particles—that interacts with each particle. Newton's third law does not always apply to particles that are under such a force.

Rocket motion

When the mass m is permitted to change, the quantitative application of momentum conservation may get a bit challenging. This is true of rockets since the majority of their bulk is made up of fuel that is ultimately discharged. Let's say that mass is expelled out from the rocket at a speed of (constant) u .¹⁵ Since u is a speed in this context, it is defined to be positive. In other words, the velocity of the expelled particles is calculated by deducting u from the rocket's velocity. Let M be the initial mass of the rocket and m be the later (changing) mass. The rocket's mass will change at a negative rate, or dm/dt . Therefore, mass is ejected at the positive rate $|dm/dt| = -dm/dt$. In other words, a negative mass (dm) is added to the rocket within a brief period of time (dt), and a positive mass (dm) is fired out the rear. It may seem ridiculous, but the toughest part of rocket motion is choosing a sign for these values and sticking with it (if you wanted, you could define dm to be positive, then remove it from the rocket's mass, and have dm get fired out the rear).

It is assumed that a certain frame of reference has been selected while discussing momentum. After all, it is necessary to measure the particle velocities in relation to a coordinate system. We shall show that there is one specific reference frame that is often favourable to utilise, although it is acceptable to use any inertial (that is, nonaccelerating) frame. Take into account a frame S and a second frame S' that travels at a constant u with relation to S (Fig. 5.12). The speed of the system of particles, given a

According to this relationship, momentum must be preserved during a collision in frame S' in order to be conserved in frame S . This is true because, relative to S' , the system's initial and final momenta in S have both grown by the same amount. So let's think about the one and

only frame when a system of particles has zero total momentum. The centre of mass frame, or CM frame, is what we have here. The CM frame is the frame S' that travels with velocity $\mathbf{v}$ if the total momentum is $\mathbf{P}$ in frame S .

The CM frame is quite practical. In this frame, physical processes are often considerably more symmetrical, which increases the transparency of the outcomes. The "zero momentum" frame is another name for the CM frame. The center of mass of the particles, however, does not move in the CM frame, which is why the term "center of mass" is often employed. Centre of mass location is determined by in contrast, we made no mention of the frame we were employing in our prior derivation of momentum conservation. Only that the frame wasn't speeding was assumed. It wouldn't equal $\mathbf{F} = m\mathbf{a}$ if it were accelerating, therefore $\mathbf{F}$ would not. In Chapter 10, we'll observe how $\mathbf{F} = m\mathbf{a}$ changes in a non-inertial frame. However, there is no need to fret in this situation here is where the pivot would be for a rigid system to balance. Since the derivative of $\mathbf{R}_{CM}$ in Eq. equals the velocity of the CM frame, it follows that the CM does not move with respect to the CM frame. Therefore, the origin of the CM frame may be decided upon as the center of mass.

In order to calculate the CM's acceleration, we may simply apply $\mathbf{F} = m\mathbf{a}$ to the system of particles at the CM by treating it as a point mass. We only need to take the external forces into account when calculating $\mathbf{F}_{total}$ since the internal forces cancel in pairs. The lab frame is the second frame that individuals often work with in addition to the CM frame. There is absolutely nothing unique about this frame. It is only the frame—assumed to be inertial—in which the problem's circumstances are stated. The "lab frame" is any inertial frame, and problem solving often requires transitioning between the lab and CM frames. For instance, if you are asked for the final response in the lab frame, you may want to convert the provided data from the lab frame to the CM frame, where things are more evident, and then transform back to the lab frame to provide the response.

Collisions

Particle collisions may be divided into two categories: elastic collisions, in which kinetic energy is preserved, and inelastic collisions, in which kinetic energy is lost. The overall energy is preserved in every collision, but in inelastic collisions, some of this energy is converted into heat (i.e., relative motion of the molecules within the particles) rather than manifesting as net translational motion. Although certain circumstances are fundamentally inelastic, as we'll see in Section 5.8, we'll focus on elastic collisions in this section. Only a little change to the process is needed for inelastic collisions if it is indicated that a particular percentage, say 20%, of the kinetic energy is lost. We just need to put down the conservation of energy and momentum equations, then solve for any variables we wish to discover, to answer any elastic collision issue. In mechanics, collisions are interactions between two or more objects that modify the velocities of each of them. The study of collisions is a crucial component of classical mechanics and is used to examine and comprehend how things move in a variety of contexts. Elastic and inelastic collisions are the two primary categories of collisions.

Algorithmic Collisions:

Kinetic energy and momentum are preserved in elastic collisions. This implies that the entire kinetic energy and total momentum of the system are preserved both before and after the collision. The reason why elastic collisions are sometimes referred to as "perfect" collisions is because no kinetic energy is wasted to internal energy from friction or item deformation. By

using the laws of momentum and kinetic energy conservation, it is possible to calculate the velocities of two objects before and after an elastic collision. In scenarios like billiard balls, perfect gases, and certain subatomic particle interactions, elastic collisions are often seen [11], [12].

CONCLUSION

Momentum is preserved in inelastic collisions, but kinetic energy is not. Some kinetic energy is converted into internal energy or deformation energy, among other forms. Due to internal effort or non-conservative forces, inelastic collisions are less "perfect" than elastic collisions. An inelastic collision might result in the items sticking together or breaking apart at differing velocities. The precise result relies on the details of the accident. Inelastic collisions are often seen in real-world scenarios, such as auto accidents, sporting events, and several industrial operations. The concepts of momentum and kinetic energy conservation are often used in the analysis of collisions in mechanics. The ultimate velocities of the items involved in the collision may be calculated using these ideas. Different equations and methods are used depending on whether the collision is elastic or inelastic. It's crucial to remember that not every collision can be classified as either entirely elastic or totally inelastic. A spectrum of possibilities exists between fully elastic and perfectly inelastic collisions, and some collisions show traits of both kinds. A thorough examination of real-world collisions must take into account a number of variables, including friction, deformation, and energy dissipation.

REFERENCES:

- [1] C. de Rham, "Massive gravity," *Living Reviews in Relativity*. 2014.
- [2] C. Sakumura, S. Bettadpur, and S. Bruinsma, "Ensemble prediction and intercomparison analysis of GRACE time-variable gravity field models," *Geophys. Res. Lett.*, 2014.
- [3] R. Koppram, E. Tomás-Pejó, C. Xiros, and L. Olsson, "Lignocellulosic ethanol production at high-gravity: Challenges and perspectives," *Trends in Biotechnology*. 2014.
- [4] C. Morley, J. Rosselló, and M. Santana-Gallego, "Gravity models for tourism demand: Theory and use," *Ann. Tour. Res.*, 2014.
- [5] C. Hirt, M. Kuhn, S. Claessens, R. Pail, K. Seitz, and T. Gruber, "Study of the Earth's short-scale gravity field using the ERTM2160 gravity model," *Comput. Geosci.*, 2014.
- [6] N. Blaser, S. I. Guskov, V. A. Entin, D. P. Wolfer, V. A. Kanevskyi, and H. P. Lipp, "Gravity anomalies without geomagnetic disturbances interfere with pigeon homing - A GPS tracking study," *J. Exp. Biol.*, 2014.
- [7] S. Carlip, "Challenges for emergent gravity," *Stud. Hist. Philos. Sci. Part B - Stud. Hist. Philos. Mod. Phys.*, 2014.
- [8] F. Sylos-Labini, F. Lacquaniti, and Y. P. Ivanenko, "Human Locomotion under Reduced Gravity Conditions: Biomechanical and Neurophysiological Considerations," *BioMed Research International*. 2014.
- [9] J. Callies, R. Ferrari, and O. Bühler, "Transition from geostrophic turbulence to inertia-gravity waves in the atmospheric energy spectrum," *Proc. Natl. Acad. Sci. U. S. A.*, 2014.

- [10] O. Carraz, C. Siemes, L. Massotti, R. Haagmans, and P. Silvestrin, “A Spaceborne Gravity Gradiometer Concept Based on Cold Atom Interferometers for Measuring Earth’s Gravity Field,” *Microgravity Sci. Technol.*, 2014.
- [11] D. Luo, F. Cai, and Z. Liu, “A new method of gravity external correction for high precision gravity measurement,” *J. Appl. Geophys.*, 2014.
- [12] K. Hasegawa, P. S. de Campos, J. L. Zeredo, and Y. Kumei, “Cineradiographic analysis of mouse postural response to alteration of gravity and jerk (gravity deceleration rate),” *Life*, 2014.

CHAPTER 7

THE ANALYSIS OF ANGULAR MOMENTUM

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ABSTRACT:

The scaling argument provides the first line, the moments of inertia simply sum (the left and right sides are duplicates that are joined at the pivot), and the parallel-axis theorem provides the second line. The right-hand sides of the first two equations are equated to produce. The desired outcome is obtained by inserting this expression into the third equation. Keep in mind that we will eventually need to employ actual, living numbers, which are introduced here through the parallel-axis theorem. Utilizing merely scaling considerations is insufficient since they only provide linear equations that are homogenous in leaving no way to determine the appropriate dimensions. You may wow your pals by declaring that you understand how to "use scaling arguments along with the parallel-axis theorem to calculate moments of inertia of objects with fractal dimension" if you've mastered this method and used it on the fractal objects. Additionally, you never know when that could be useful.

KEYWORDS

Axis, Force, Mass, Objects, Parallel

INTRODUCTION

According to a certain origin, a point mass's angular momentum is defined by:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

The total $\mathbf{L}$ for a group of particles is just the sum of the $\mathbf{L}$ s of all the particles. The vector $\mathbf{r} \times \mathbf{p}$ has several interesting features, making it an excellent object to study. One of these is the conservation rule discussed in, which gave us the opportunity to introduce the concept of "effective potential" Later on in this chapter, we will also introduce the idea of torque, which is represented by the fundamental equation $\tau = d\mathbf{L}/dt$ (corresponding to Newton's $\mathbf{F} = d\mathbf{p}/dt$ law).

In the world, angular momentum issues may be classified into two categories. We will see that it is necessary to find how $\mathbf{L}$ varies with time since, as we will see, the answer to any rotating issue always boils down to utilising $\tau = d\mathbf{L}/dt$. And since $\mathbf{L}$ is a vector, it may vary due to (1) a change in length or (2) a change in direction (or a combination of both effects). In other words, if we write $\mathbf{L} = L\hat{\mathbf{L}}$, where $\hat{\mathbf{L}}$ is the unit vector in the $\mathbf{L}$ direction, then $\mathbf{L}$ may vary depending on whether L or $\hat{\mathbf{L}}$ changes, or both [1], [2].

The first of these examples that of constant $\mathbf{L}$, is the most straightforward. Think of the genesis as the centre of a rotating record. Every term in the sum has the condition that the vector $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ is perpendicular to the record. The record will accelerate in a precise manner that we will quickly decide whether we apply a tangential force in the right direction. Nothing strange is occurring in this situation. The record moves more quickly if we apply pressure. $\mathbf{L}$ continues to point in the same general direction, but now with a bigger magnitude. In fact, we may fully disregard the notion that $\mathbf{L}$ is a vector in this kind of situation. Everything will be OK if we can simply handle its magnitude L . The focus of this chapter is on this first example. However, the second instance of $\mathbf{L}$ changing direction might be quite perplexing.

The topic of the next chapter will be whirling tops and other items that spin, which have a propensity to make people's heads spin as well. The whole idea in these instances is that L is truly a vector. And in order to understand what is happening, we really need to see the situation in three dimensions, unlike in the constant- L scenario.

The straightforward provides the value of the angular momentum of a point mass. But we need to understand how to compute the angular momentum of an extended object in order to deal with setups in the real world, which inevitably include numerous particles [3], [4].

Pancake object in the x-y plane

Consider a flat, rigid body moving arbitrarily in the x-y plane (both translating and spinning);. What is the body's angular momentum with relation to the coordinate system's origin? If we consider the body to be made up of m_i mass particles, then the angular momentum of the whole body is equal to the total of the m_i mass particles' individual angular momenta, or $L_i = r_i p_i$. The entire angular momentum is hence instead of a sum, we would have an integral for a continuous distribution of mass. The masses' positions and momenta affect L . The momenta themselves depend on how quickly the body is rotating and translating. Finding the relationship between L and the distribution and mobility of the component masses is our objective in this case. We will demonstrate how the outcome will specifically involve the body's geometry.

Only things that move in the x-y plane and resemble pancakes will be discussed in this section. In addition to determining an equation for the kinetic energy, we will compute L in relation to the origin. Note that the vector $L = r \times p$ always points in the direction of z since the r and p of all the masses in our pancake-like objects always reside in the x-y plane. This fact, as previously mentioned, is what makes these pancake cases simple to handle. L varies exclusively in proportion to its length, not in any other way. So the equation $\frac{dL}{dt}$ will have a simple form when we finally come to it. Before we examine universal motion in the x-y plane, let's first examine a specific situation.

The distinction between these two situations and the one between the two fundamental $F = dp/dt$ cases is basically the same. The vector p may fluctuate due of its fluctuating magnitude, in which case $F = ma$ holds true (assumes m is constant). Alternatively, p could alter due to a change in direction, in which case $F = mv^2/r$, the centripetal-acceleration assertion, applies. (Alternatively, a mixture of these consequences could occur.) The first scenario looks a little more logical than the second. Remember, L is defined relative to a specified origin, since it contains the vector r in it. Therefore, asking what L is without mentioning the origin we've picked is meaningless [5], [6].

X-Y plane motion in general

How do we handle x-y plane general motion? Because the different components of mass don't revolve about the origin during the motion, when the object is simultaneously translating and spinning, we are unable to write $v = r\omega$ as we did before. Writing the angular momentum, L , and the kinetic energy, T , in terms of the center-of-mass (CM) coordinates and the coordinates relative to the CM, turns out to be quite useful. We now demonstrate how writing the formulas for L and T in this manner results in some extremely attractive forms. Let $R = (X, Y)$ represent the location of the CM in relation to a fixed origin. Let $r = (x, y)$ represent the location of a particular point in relation to the CM. Then, $r = R + r$ represents the location of the specified point in relation to the fixed origin. Let the CM's velocity be V and the relative velocity to the CM be v . So, $v = V + v$. Let the body spin with angular speed

$\omega \times$ about the CM (around an instantaneous axis parallel to the z axis, such that the pancake stays on the x - y plane at all times).

DISCUSSION

Think about the unique situation in which the body spins around the CM at the same pace as the CM rotates around the origin. One way to do this is to glue a stick across the pancake and pivot one end of the stick at the origin; for an example, see Fig. 8.5. In this particular instance, we have a simplified scenario in which all points in the pancake circle the origin. Let the angle of their rotation be θ . A spool with a very tiny inner radius rolling down a narrow plane with just its inner "axle" rolling on the plane or lengthy spokes sticking out of the cylinder and passing through a deep groove in the plane may both do this. Alternatively said, the moment of inertia at the origin is the parallel-axis theorem is as follows. It states that once the moment of inertia of an object around an axis passing through the CM (specifically I_{CM}) has been calculated, adding on MR² will allow you to calculate the moment of inertia around a parallel axis. R is the distance between the two axes, and M is the mass of the object. The parallel-axis theorem only holds true with the CM and not with any other point, since it is a particular case [7], [8].

Non-planar Objects

We limited the subject to pancake objects in the x - y plane. However, almost all of the conclusions we reached apply to non-planar objects, given that the rotation's axis is parallel to the z axis and that we are only interested in L_z and not L_x or L_y . So let's exclude the pancake hypothesis and proceed with the outcomes we found above. Consider an object that is initially spinning around the z axis. Allow for z -direction extension of the object and provide the L_z for each pancake when the object is divided into pancakes parallel to the x - y plane. Additionally, we may deduce that the I_z of the whole object is equal to the total of the I_z of all the pancakes since the L_z of the entire object is equal to the sum of the L_z of all the pancakes. It doesn't matter if the pancakes' z values vary. Consequently, if an item is rotating around the z axis, then. The analysis in this chapter is not completely generic even provides the L_z for any item because we are constraining the axis of rotation to be the (fixed) z axis and even with this constraint, an object outside the x - y plane could still be considered x and y components of L that are nonzero, but we only discovered the z component. This second observation is odd yet accurate.

Because we can calculate the overall T by summing the T 's of all the pancake slices, continues to provide the T for a nonplanar object rotating along the z axis also apply for a nonplanar object when the CM is translating and the object is rotating around it (or, more accurately, along an axis that is parallel to the z axis and passes through the CM). In reality, these two equations remain true regardless of which way the CM's velocity V points on this chapter, however, we'll assume that all velocities are on the x - y plane. Finally, a nonplanar object is nonetheless subject to the parallel-axis theorem. But the perpendicular-axis theorem is invalid. The only time the planar assumption is necessary is in this situation [9], [10].

Clever Trick

It is feasible to determine the moment of inertia for certain objects with specific symmetries without doing any integral calculations. The parallel-axis theorem and a scaling argument are the only things we need. This method will be shown by locating the I for a stick near its centre. Other applications may be found in the chapter's difficulties.

The key idea in the current case is to compare the I for a stick of length L with the I for a stick of length $2L$. The former is eight times the later, according to a simple scaling argument. This is accurate since the integral $\int x^2 dm = \int x^2 dx$ has three x powers (the dx does count). A factor of $2^3 = 8$ results from changing the variables to $x = 2y$. A matching component in the bigger stick will be twice as enormous and far away from the axis if we envisage growing the smaller stick into the larger one. As a result, the integral $\int x^2 dm$ grows by a factor of $2^2 \cdot 2 = 8$.

Force

We shall now demonstrate that, under certain circumstances (described below), the torque, which we refer to as, is equal to the rate of change of angular momentum. Therefore, $\tau = dL/dt$. This is the rotating equivalent of the linear momentum equation $F = dp/dt$. The fundamental concept is simple, but there are two nuanced problems. One deals with the forces that operate within a group of particles. The second is concerned with the origin's potential acceleration (the origin is the location from which torque and angular momentum are derived). To keep things organised, we'll address three examples that become progressively more complex as we demonstrate the basic conclusion. We may apply the outcome from this chapter's derivation of $\tau = dL/dt$ for entirely generic motion in the next chapter. If the axis of rotation is parallel to the z axis, you may provide a more precise proof of the statement that $\tau = dL/dt$. However, because the general proof isn't much more challenging, we'll deliver it in this chapter and put an end.

A change in an object's motion or state of rest may be brought about by an effect, which is referred to as force in physics. It has both a direction and a magnitude since it is a vector quantity. An item may be pushed or pulled by a force, which can work in a variety of ways to cause an object to accelerate, decelerate, or change direction.

Force Concept: Importance

Because it serves as the foundation for understanding how things interact with one another and with their environment, the idea of force is of utmost significance in physics. The idea of force is essential to understanding and forecasting these events, whether we are thinking about the motion of planets in the solar system, the behaviour of a bouncing ball, the functioning of machines, or the interactions of subatomic particles. This article will examine the evolution of the idea of force throughout history, how it appears in both classical and contemporary physics, and how it has enormous effects on how we see the cosmos.

Ancient Theories of Force

Ancient civilizations have profound historical origins that go back to the idea of force. Aristotle is one of the first philosophers to put out concepts regarding force and motion. According to Aristotle, things naturally inclined to come to rest and needed a power to keep them moving. However, the mathematical rigour and accuracy that would eventually distinguish the scientific study of force were absent from these early theories.

Galileo and Kepler: The Fathers of Modern Physics

Galileo Galilei and Johannes Kepler's pioneering research in the 17th century marked the beginning of the current study of force. An item in motion will stay in motion until acted upon by an external force, according to the rule of inertia, which was developed as a result of Galileo's studies with inclined planes and falling objects. Further understanding of the

function of force in celestial motion was supplied by Kepler's equations of planetary motion, which were based on thorough measurements of the planets.

Newton's Principles of Motion

The publication of Sir Isaac Newton's "Mathematical Principles of Natural Philosophy" (commonly referred to as the "Principia") in 1687 marked the most important turning point in the evolution of the idea of force. Our notion of force has been fundamentally altered by Newton's laws of motion, which also served as the basis for classical mechanics.

First Law of Newton: The Law of Inertia

According to Newton's first rule of motion, unless acted upon by an outside force, an object will stay at rest or in uniform motion along a straight path. The idea of inertia, or an object's propensity to resist changes in its state of motion, was first presented by this rule. The fundamental relationship between force and motion is known as Newton's second law. The link between force, mass, and acceleration is quantified by Newton's second law of motion. It asserts that an object's acceleration is inversely proportional to its mass and directly proportional to the net force applied on it. The equation: is a frequent way to represent this rule.

$F=ma$, F is the object's force (measured in newtons, N), m is the object's mass (measured in kilogrammes, kg), and a is the object's acceleration (measured in metres per second squared, m/s^2). This fundamental rule of classical mechanics enables us to determine how a force affects an object's motion.

Action and Reaction: The Third Law of Newton

There is an equal and opposite response to every action, according to Newton's third rule of motion. In other words, if object A applies a force on object B, object B will apply a force that is equal to and opposing to that applied by object A. In order to comprehend how things interact with one another, it is essential to understand why forces usually occur in pairs.

The Development of Classical Physics

Newton's laws of motion provide a comprehensive foundation for comprehending how things respond to forces. This brand-new branch of physics, known as classical mechanics, served as the foundation for later advances in both physics and engineering. It made it possible for engineers and scientists to accurately anticipate and regulate the motion of things, which sparked technological developments that helped to construct the contemporary world.

Force of Gravitation

The force of gravity is one of the most well-known forces in classical mechanics. It is the force that draws mass-containing things together. The mass of the objects and their separation from one another determine the gravitational force's intensity. One of the basic forces of nature is the force of gravitation, sometimes known as gravity. It is a fundamental idea in physics and is what causes the attraction between all things with mass. The following list of fundamental concepts and equations that define gravity is provided:

Newton's Law of Universal Gravitation (universal gravitation equation):

The law of universal gravitation was created in the 17th century by Sir Isaac Newton. According to this rule, any mass at a given place is drawn to every other mass at that same

position by a force operating along the line that cuts both points. The following equation yields the force's magnitude: The distance r (measured in metres, m) between the centres of the two objects. The attraction between two mass-containing objects may be calculated using this equation. The force diminishes as the distance between the items widens because it obeys the inverse square rule.

Relativistic Field:

To clarify how each mass-containing item generates a zone in space where the gravitational force acts on other objects, the idea of a gravitational field is presented. The strength of the gravitational field (The gravitational force (g) is measured at a particular location in space. At that time, m) U is the energy corresponding to an object's location in relation to another in a gravitational field. It is based on the spacing and mass of the objects. Gravitational potential energy is calculated as follows: Because gravitational potential energy is the work done by the gravitational pull to bring two things from an infinite separation to their present separation, it is a negative quantity.

Weight:

The gravitational pull on an item is measured by its weight. It serves as a gauge for the gravitational attraction that pulls an item downward. Escape velocity, which establishes the bare minimum speed necessary for spacecraft to escape a celestial body's influence, is crucial for space exploration. A basic and pervasive force in the cosmos is gravity. It controls how celestial bodies move, affects how things behave on Earth, and has significant ramifications for astronomy, astrophysics, and our comprehension of the universe. A basic tenet of physics, known as Newton's Law of Universal Gravitation, defines the gravitational interaction between two mass-containing objects. Sir Isaac Newton first proposed it in 1687. According to this principle, every particle of matter in the universe pulls away from every other particle with a force that is exactly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. This rule may be expressed mathematically as follows: The distance r (measured in metres, m) between the centres of the two objects.

Important details about Newton's Law of Universal Gravitation:

The force of gravity decreases with the square of the separation between two objects, according to the Inverse Square Law. This indicates that the force of gravity is decreased to one-fourth of its initial magnitude if the distance between two objects is doubled.

Relationship to Mass:

The gravitational force is inversely related to the sum of the masses of the two objects. This implies that as the mass of one or both objects grows, so does the force of gravity [11], [12].

CONCLUSION

Regardless of an object's size or make-up, this rule is true for all things in the cosmos. It controls the velocity of quantum particles as well as planets, stars, galaxies, and other celestial objects. The gravitational force is always attracting, which means it draws items together. It operates along the axis that connects the two objects' centers. Newton calculated the mass of the Earth using this formula, and he also offered a technique for figuring out the masses of other celestial bodies. Newton's Law of Universal Gravitation, a cornerstone of celestial mechanics, is essential to understanding the motion of celestial objects, the orbits of

planets, and many other astronomical phenomena. Newton's law, which accurately describes gravity in the majority of scenarios seen on a daily basis, was generalized by Albert Einstein in the early 20th century. Gravity is better understood by Einstein's theory, especially in harsh conditions close to huge objects. Newton's Law of Universal Gravitation continues to be a useful and trustworthy tool for the majority of daily applications requiring gravity, including space exploration, satellite dynamics, and many engineering calculations on Earth, despite the development of Einstein's theory.

REFERENCES:

- [1] E. Karimi, S. A. Schulz, I. De Leon, H. Qassim, J. Upham, and R. W. Boyd, "Generating optical orbital angular momentum at visible wavelengths using a plasmonic metasurface," *Light Sci. Appl.*, 2014.
- [2] F. Bouchard, H. Mand, M. Mirhosseini, E. Karimi, and R. W. Boyd, "Achromatic orbital angular momentum generator," *New J. Phys.*, 2014.
- [3] N. Bergeard *et al.*, "Ultrafast angular momentum transfer in multisublattice ferrimagnets," *Nat. Commun.*, 2014.
- [4] P. Martelli and M. Martinelli, "Representation and evolution of the angular momentum of the light," *IEEE Photonics J.*, 2014.
- [5] A. Fleischer, O. Kfir, T. Diskin, P. Sidorenko, and O. Cohen, "Spin angular momentum and tunable polarization in high-harmonic generation," *Nat. Photonics*, 2014.
- [6] A. K. Silverman, R. R. Neptune, E. H. Sinitski, and J. M. Wilken, "Whole-body angular momentum during stair ascent and descent," *Gait Posture*, 2014.
- [7] L. Adhikari and M. Burkardt, "Distribution of angular momentum in the transverse plane," *Nucl. Phys. B - Proc. Suppl.*, 2014.
- [8] J. W. Kim, M. Vomir, and J. Y. Bigot, "Controlling the spins angular momentum in ferromagnets with sequences of picosecond acoustic pulses," *Sci. Rep.*, 2014.
- [9] W. T. Buono, L. F. C. Moraes, J. A. O. Huguenin, C. E. R. Souza, and A. Z. Khoury, "Arbitrary orbital angular momentum addition in second harmonic generation," *New J. Phys.*, 2014.
- [10] P. H. Lauritzen, J. T. Bacmeister, T. Dubos, S. Lebonnois, and M. A. Taylor, "Held-Suarez simulations with the Community Atmosphere Model Spectral Element (CAM-SE) dynamical core: A global axial angular momentum analysis using Eulerian and floating Lagrangian vertical coordinates," *J. Adv. Model. Earth Syst.*, 2014.
- [11] Wei Gongxiang, Liu Xiaojuan, Liu Yunyan, and Fu Shenggui, "Spin and Orbital Angular Momentum of Light," *Laser Optoelectron. Prog.*, 2014.
- [12] K. Yamane, Z. Yang, Y. Toda, and R. Morita, "Frequency-resolved measurement of the orbital angular momentum spectrum of femtosecond ultra-broadband optical-vortex pulses based on field reconstruction," *New J. Phys.*, 2014.

CHAPTER 8

IMPACT OF COLLISIONS

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ABSTRACT:

We studied collisions of point particles or nonrotating objects. There, linear momentum conservation and energy conservation (if the impact was elastic) were the key components that allowed us to solve issues. Now that we have the concept of conservation of angular momentum, we can expand our study of collisions to include those involving spinning objects. The added degree of freedom in the rotation makes up for the extra fact of L conservation. We will thus continue to have the same amount of equations and unknowns as long as the issue is structured correctly. Only an elastic collision may be utilized to apply conservation of energy to it. However, given that the system is isolated, conservation of angular momentum is equivalent to conservation of linear momentum in that it may always be exploited (see the comment below). However, since we must choose an origin before moving forward, conservation of L differs somewhat from conservation of p . We must choose our origin to be either a fixed point or the system's CM in light of the three requirements that requires to be true.

KEYWORDS:

Angular, Energy, Mass, Momentum, Motion

INTRODUCTION

We have no authority to assert that dL/dt equals zero only because the torque is zero (as it is for an isolated system) if we arbitrarily choose an accelerating point since $= dL/dt$ does not hold. It is simple to make the mistake of choosing an accelerating point as your origin while solving collision situations. For instance, you may decide to pick the origin as the middle of a stick. However, the center will accelerate if another item strikes the stick, making it a poor candidate for the origin. In terms of conservation, E differs from p and L in that energy may be concealed as heat in the microscopic motion of body molecules. This motion is made up of tiny vibrations that have high speeds but modest amplitudes. These vibrations' energy may be sufficiently strong to be of the same order of magnitude as the system's total energy. But it seems that energy is wasted since the vibrations are too tiny to notice. Even if they are too little to see, you can still feel heat from them with your touch despite their size [1], [2].

However, linear momentum cannot be concealed. There is no getting around the fact that if an object—not necessarily one that is rigid—has nonzero momentum, it must be moving as a whole. In other words, since $P = MV_{CM}$, we can see that if P is nonzero, V_{CM} must likewise be. There is no way for the motion to be concealed on a tiny size, thus it must be on a macroscopic scale. However, there is one extremely widespread phenomenon—magnetism—that (in a sense) is an exception to all three of the aforementioned statements. Despite not being an angular momentum, magnetism does originate from the "circular" motion of electrons around atom nuclei. (In general, it actually comes more from the "spin" of the electrons than their orbital motion around the nuclei, but let's not worry about that here. Let's just work in a rough classical approximation.) Electrons move in tiny little correlated loops throughout a magnetic material. First, the magnetic field includes the electric charge e , and

this is big enough (on the scale of things) to balance out the smallness of the r component. This allows us to avoid the preceding three lines of reasoning. Second, it is quite simple to get the electrons moving in correlated circles by means of magnetic forces; there is no need to smash things together (the actual angular momentum of the electrons in a magnetic material is negligible because their mass is so small). Third, electrons may freely travel about in atoms in classically defined little circles without tearing anything apart. It's crucial to keep in mind that you are free to choose your origin from among the legitimate options of fixed spots or the CM. You should make use of this flexibility as it is often the case that one option is preferable to others (in that it simplifies the computations). Let's use two instances. An elastic collision happens first, followed by an inelastic one [3], [4].

Angular impulse

The temporal integral of the force applied to an object was our definition of the impulse, or I . The net change in linear momentum is what we refer to as the impulse according to Newton's second law, $F = dp/dt$. The torque imparted to an object is now defined as the time integral of the angular impulse, I . The angular impulse is the net result of $= dL/dt$ angular momentum shift. These are only definitions with no actual substance. The following is when physics enters the picture. Consider an instance in which the origin around which (t) is computed is always at the same location with respect to $F(t)$ (of course, this origin must be lawful). Let R be this position [5], [6].

DISCUSSION

The relationship between the net changes in L is shown and p rather than their separate values. Even though $F(t)$ is changing arbitrarily over time and we are unsure of what L and p are, we still understand that they are connected. We often don't need to be concerned with the cross product as the lever arm R is parallel to the momentum change p . When this occurs, we have additionally, the item often begins at rest, so we don't need to worry about the's. The application of angular impulse and Eq is shown in the example that follows.

Enormous Pulley

Think about the Atwood's device the pulley is a uniform disc with a mass of m and a radius of r , and the masses are m and $2m$. Since the string has no mass, it does not slide in relation to the pulley. Identify the mass acceleration. Use energy conservation.

Departure from the sphere

On top of a fixed sphere, a ball with moment of inertia mr^2 is resting. The ball and the sphere rub against one other. With a little kick, the ball slides down the hill without skidding. What point does the ball lose touch with the sphere if r is much less than the radius of the sphere? If the ball's size is more than or equal to that of the sphere, how does your response change?

Sliding ladder

Standing on a frictionless floor and leaning against a frictionless wall is a ladder of length four and uniform mass density. With its bottom end a microscopic distance from the wall, it is first kept still and immobile. When it is released, the top end slides down the wall and the bottom end glides away from the wall. What is the centre of mass's horizontal component of velocity when it breaks away from the wall?

Rectangle with a lean

A stationary cylinder with radius R is topped by a rectangle with dimensions of $2a$ in height and $2b$ in breadth. I is the rectangle's moment of inertia around its centre. After receiving a little kick, the rectangle "rolls" on the cylinder without sliding. Find the motion equation for the rectangle's tilt angle. What circumstances will the rectangle oscillate back and forth under and will it fall off the cylinder? Discover the oscillations' frequency.

Mass in a tube

A tube with a mass M and a length of 4 may freely swing around a pivot. This end of the (frictionless) tube has a mass m . After holding the tube horizontal, it is let go (see Fig. 8.24). Let θ denote the tube's angle with respect to the horizontal and x denote the length of the tube that the mass has traversed. Write the Euler-Lagrange equations for θ and x in terms of θ and $x/4$ (the percentage of the length of the tube), then get the equations for θ and x [7], [8].

Calculating moments of inertia

Find the moments of inertia for the following fractal objects in the spirit of Section 8.3.2. Take caution while scaling the bulk. (a) Cut off the centre part of a stick that is 4 length. Next, take each of the last two pieces and cut off the centre third. Next, take the centre third out of each of the four pieces that are left. Assume that the finished item has mass m and that its axis runs through its centre and is parallel to the stick. For those who like this kind of stuff, this item is the Cantor set. Since it has no length, the remaining material has an infinitely high density. Simply visualise the aforementioned loop occurring only, say, a million times if you suddenly find yourself having an aversion to point masses with infinite density.

Remove the "middle" triangle, which makes up $1/4$ of an equilateral triangle with a side of 4 , from the triangle. After that, eliminate the "middle" triangle from each of the three triangles that are still present. Assuming the finished item has mass m and an axis that passes through the centre and is parallel to the plane.

Take a square from side 4 and eliminate the "middle" square, which occupies $1/9$ of the square. After that, eliminate the "middle" square from each of the eight squares that are left. Assuming the finished item has mass m and an axis that passes through the centre and is parallel to the plane

Torque Due To Internal Forces

Let F_{int} be the force on the i th particle caused by all the other particles in a group of particles with locations r_i . Considering the force between Use Newton's third law to demonstrate that $\sum_i F_{int} = 0$ when the direction of any two particles is down the line between them.

Taking away a support

(A) A uniform rod with a mass of m and a length of 4 rests on supports at both ends. Quickly remove the right support (see Fig. 8.30). What follows next in terms of the left support's force?

(b) A rod with a moment of inertia of mr^2 and a length $2r$ sits atop two supports, each of which is at d distances from the center. Quickly remove the right support (see Fig. 8.30). What follows next in terms of the left support's force?

Dropping stick

A stick of mass m and length 4 has a massless stick of length b with one end hinged on a support and the other end glued perpendicular to the center. What is the initial acceleration of the CM if the two sticks are held in a horizontal plane (see Fig. 8.31) and subsequently released? (b) What is the initial acceleration of the CM if the two sticks are held in a vertical plane (see Fig. 8.31) and subsequently released?

Removing a Cylinder

The cylinder goes immediately to the right in Exercise 8.50 below. The lack of transverse motion results from the fact that the two string segments only pull in the correct direction and cannot, thus, produce a transverse force. Explicitly demonstrate this outcome once again incorporating the force of the string on the cylinder across the area of contact. It will be helpful to remember the $N = Td$ result from the "Rope wrapped around a pole"

Circular motion cylinders

On the inside surface of another hollow cylinder with mass M_2 and radius R_2 , a hollow cylinder with mass M_1 and radius R_1 rolls without sliding. Suppose $R_1 < R_2$ exists. The bigger cylinder is free to revolve around its axis since both axes are horizontal. What frequency do tiny oscillations occur at?

Increasing the string's length

As shown in Fig., a mass swings in a horizontal circle when suspended from a massless thread. The string's length is then very gradually lengthened or shortened. Let's define, 4 , r , and h as illustrated. How does r rely on 4 in assuming that is relatively small? How does h rely on 4 if we assume that is extremely near to $1/2$? a triangle made up of cylinders The triangle in Fig. is formed by three identical cylinders with moments of inertia. For the next two situations, determine the upper cylinder's starting downward acceleration. Which situation has the greater acceleration? There is no friction between any of the cylinders, but there is friction between the bottom two cylinders and the ground There is friction inside the cylinders but none between the bottom two cylinders and the ground.

Chimney falling

A chimney starts off upright. It receives a little kick, and it collapses. Where along its length is the most probable place for it to break? Use the following simple two-dimensional chimney model to solve the given issue. Assume that the chimney is made of stacked boards, and that each board is fastened to the two boards next to it.

Stick being struck by ball

A stick with a moment of inertia $I = \frac{1}{2}m4^2$ (relative to its centre, which is its CM) collides with a ball of mass M . At first, the ball is moving perpendicular to the stick at speed V_0 . A distance of d from the centre is where the ball collides with the stick (see Fig. 8.36). It is an elastic collision. Find the stick's resultant translational and rotational speeds as well as the ball's consequent speed.

A "ball and stick" principle

Think about how Problem 8.18 is put up. Clearly demonstrate that the relative speeds of the ball and the stick's point of contact are the same before and just after a collision. (This

outcome is comparable to Theorem 5.3 in Section 5.7.1's "relative speed" conclusion for a 1-D collision.)

Angle Impulse

$I = (2/5)mR^2$ is used to launch a ball of radius R into the air. It revolves around an axis that is parallel to the motion's vertical plane. This is the "x-y plane." The ball makes contact with the floor and bounces off it without sliding. Assume that the vertical v_y magnitude is the same before and after the bounce and that the impact is elastic. By demonstrating the relationship between v_x and v after the bounce,

Bouncing off a bump

A ball with a radius of R and an inertia moment of $I = (2/5)mR^2$ rolls at a speed of V_0 without sliding. When it comes across a step, it rolls up over it. If the ball adheres to the corner of step for a little period of time (until the ball's centre is just above the corner). Show that V_0 must be satisfied in order for the ball to ascend over the step. One morning, after buttering your toast (assumed to be a uniform rigid square with a side length of 4 for the purpose of having a workable problem), you unintentionally drop it from a height H above a counter, which is itself h above the ground. When the toast falls, one edge just barely contacts the surface (elastically), spinning the bread since it is aligned "parallel" to the counter. What value of H produces the terrible situation where the toast makes half a rotation and lands buttered side down on the ground when expressed in terms of h and 4? 4 has a unique meaning in terms of h [9], [10].

Rolling To Sliding

On a horizontal surface with friction, a ball initially glides without turning. Moment of inertia around the center is $I = mR^2$, and starting speed is V_0 .

- Determine the speed at which the ball starts to roll without sliding without knowing anything about the makeup of the friction force. Find the kinetic energy that was lost while sliding as well.
- Now take into account the unique situation when the coefficient of kinetic friction is, regardless of location. When and how far away does the ball start to roll without slipping? Check that the energy loss determined in component (a) matches the work produced by friction.

Think of a group of stiff sticks with lengths $2r$, masses m_i , and moments of inertia $m_i r^2$ with m_1, m_2, m_3 . Each stick has a CM in the middle of it. As seen in Fig., the sticks are positioned on a horizontal, frictionless surface. The ends are very slightly spaced apart in the x direction and somewhat overlapped in the y direction.

A sudden blow (as illustrated) is delivered to the first, heaviest stick, causing it to translate and spin. After the first stick, the second stick strikes the third stick, and so on. Count on all collisions being elastic. The speed of the n th stick will either (1) approach zero, (2) approach infinity, or (3) approach infinity depending on the size be unrelated to n , because n . A uniform stick corresponds to the specific value of that, in this case, corresponds to the third of these three scenarios: $= 1/3$ [11], [12].

CONCLUSION

Things are a bit difficult when dealing with angular momentum. For similar reasoning to those in the case of hidden linear momentum, if an object is rigid, it cannot contain hidden angular momentum; because $L = I\omega$, we can show that if L is nonzero, then ω is likewise. However, it turns out that an object might potentially have hidden angular momentum in microscopic motion if it isn't stiff (for example, think of a gas of particles), even though in reality it would be too tiny to observe. Small whirling zones may produce this concealed angular momentum across the system. In contrast to linear momentum, angular momentum may exist in the absence of any overall motion. This small whirling motion thus resembles the minuscule vibrational motion that produces hidden energy. But there are three key distinctions. First, if the whirling motion occurs on a tiny scale, the r in $L = mrv$ is very small, which results in a negligible L . Due to the fact that r is not a component of the vibrational energy, this reasoning is invalid for E . Rather, it simply involves v in the form of $mv^2/2$, therefore it may grow to be substantial. Second, unlike the readily begun random linear motion that generates heat energy, the circular motion of many tiny swirls cannot be initiated by collision; you must just smash two objects together. Thirdly, if the item is solid, the molecules can readily vibrate but cannot spin endlessly since doing so would cause the links between nearby molecules to be severed. The problem is that all coordinates in vibrational motion stay tiny, however they do not in rotating motion since ultimately becomes larger

REFERENCES:

- [1] A. M. Lozen, J. Pace, N. Yoganandan, F. A. Pintar, and J. F. Cusick, "Unilateral atlanto-axial fractures in near side impact collisions: An under recognized entity in cervical trauma," *J. Craniovertebr. Junction Spine*, 2014.
- [2] R. A. Ibrahim, "Recent advances in vibro-impact dynamics and collision of ocean vessels," *J. Sound Vib.*, 2014.
- [3] K. Tsunematsu, B. Chopard, J. L. Falcone, and C. Bonadonna, "A numerical model of ballistic transport with collisions in a volcanic setting," *Comput. Geosci.*, 2014.
- [4] Y. Y. Tay, C. S. Lim, and H. M. Lankarani, "A finite element analysis of high-energy absorption cellular materials in enhancing passive safety of road vehicles in side-impact accidents," *Int. J. Crashworthiness*, 2014.
- [5] J. Everaert, "Collision risk and micro-avoidance rates of birds with wind turbines in Flanders," *Bird Study*, 2014.
- [6] V. Vashisht, "Enhancing Software Process Management through Control Charts," *J. Softw. Eng. Appl.*, 2014.
- [7] Y. G. Li, T. N. Chen, X. P. Wang, K. P. Yu, and C. Zhang, "Theoretical analysis and numerical simulation for impact noise due to collision of two cylinders," *Zhendong yu Chongji/Journal Vib. Shock*, 2014.
- [8] H. Nagaoka, S. Takasawa, A. M. Nakamura, and K. Sengen, "Degree of impactor fragmentation under collision with a regolith surface-Laboratory impact experiments of rock projectiles," *Meteorit. Planet. Sci.*, 2014.
- [9] A. T. Marques *et al.*, "Understanding bird collisions at wind farms: An updated review on the causes and possible mitigation strategies," *Biological Conservation*. 2014.

- [10] M. F. R. Grieve, C. A. Ramsbottom, C. E. Hudson, and F. P. Keenan, “Electron-impact excitation collision strengths and theoretical line intensities for transitions in S iii,” *Astrophys. J.*, 2014.
- [11] N. Zhu, Y. Kim, and T. El-Korchi, “Nonlinear analysis of bridge piers under high impact vehicle collision,” in *Structures Congress 2014 - Proceedings of the 2014 Structures Congress*, 2014.
- [12] A. Abdelsamie, A. E. Fard, T. Oster, and D. Thévenin, “Impact of the collision model for fully resolved particles interacting in a fluid,” in *American Society of Mechanical Engineers, Fluids Engineering Division (Publication) FEDSM*, 2014.

CHAPTER 9

FORM OF GENERAL MOTION

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ABSTRACT:

Rotations are sometimes difficult to visualize since they typically include three dimensions. It may not work to make a crude sketch on some paper. This makes the chapter one of the more challenging ones in the book. But to help you get started, the next few pages provide a few definitions and useful theorems. This first theorem explains the fundamental structure of any motion. Although you may think it's clear, proving it might be a bit difficult. It is possible to express the motion of the body as the combination of the translational x motion of P and some additional motion relative to P . The amounts of coordinates are additive. The last motion must be shown to be a rotation. This makes sense and is supported by the fact that the body is stiff. To put it another way, a person at rest in relation to a frame whose origin is P and whose axes are parallel to the axes of the fixed-frame perceives the body rotating about some axis via P . The distances between all locations remain constant. This theorem would not be true if the body weren't rigid. Consider a fixed, spherical shell that is centered at P in the body to be exact.

KEYWORDS:

Angular motion, Axis, General motion, Rotational, Velocity

INTRODUCTION

We need simply look at what happens to the sphere since the motion of the points on it totally determines the motion of the body. The points on the sphere must always stay the same radial distance from P since the rigid body preserves distances. In light of the fact that we are considering motion with respect to P , the issue is now limited to the following: What kind of transformation may a rigid spherical undergo into itself? Any such transformation, according to our argument, has the characteristic that there are two points that finish up where they began.² Given that distances are preserved, if one point ends up where it started, the diametrically opposite point must also end up there in order to maintain the distance of a diameter [1], [2].

If this assertion is accurate, we are done since a given point travels in just one direction for an infinitesimal transformation because there isn't enough time to do any turning. Therefore, a point that returns to its original location must have stayed fixed throughout all (infinitesimal) time. Because distances are maintained, all the locations on the diameter connecting the two fixed points must likewise have stayed fixed the whole period. This rotation around this axis is what is left. This assertion about "two points ending up where they started" is extremely plausible, but difficult to demonstrate. This claim has been left as a problem (Problem 9.2) because claims with these characteristics are always interesting to consider. On your own, try to find a solution [3], [4].

The vector of angular velocity

The angular velocity vector,, which is referred to as the vector pointing along the axis of rotation and whose magnitude equals since we are only interested in what occurs at a certain

moment in time, we are only concerned with infinitesimal transformations, even though this claim holds true for every transformation of a rigid sphere into itself.

The rotational speed.

The right-hand rule states that if you curl your right-hand fingers in the direction of the spin, then your thumb points in the direction of, which allows you to choose between the two potential orientations along the axis. For instance, a spinning record has a through its centre that is perpendicular to the record with a magnitude equal to the angular speed,. The points that (immediately) remain stationary are those that are on the axis of rotation. The points that were formerly on the axis may now be moving, since the direction of may vary with time. As long as you use the left-hand rule consistently, you may deviate from convention and decide. The orientation of will be the reverse, but because isn't actually physical, it doesn't matter. No matter which hand you employ, any physical conclusion will be the same. Your right hand will serve as a tool when you learn vectors in school. But if you look in a mirror, you'll see more clearly [5], [6].

The left-handed rule applies here as well. A feature of three dimensions is the ability to express a rotation by giving a vector. There wouldn't be a rotation if we just existed in one dimension. All rotations would occur on that plane if we were in a two-dimensional world, therefore we could just state a rotation's speed,, to identify it. on three dimensions, rotations occur on three separate planes. And for ease of reference, we decide to identify them according to the angular velocities in each plane and the directions orthogonal to these planes. If we were to exist in four dimensions, rotations would be possible in four and six planes, necessitating the labelling of rotations with six planes and six angular speeds. Be aware that a four components, four-dimensional vector would not work. ♣

Easily generates the velocity of each point in the spinning object in addition to designating the spots that are momentarily immobile. In this chapter, we will normally assume that the axis of rotation passes through the origin, unless explicitly indicated differently. The following theorem is what follows. Imagine a moment when the mass is in the y-z plane. The mass is then moving with a velocity in the x direction. As a result, the particle undoubtedly possesses angular momentum along the y and z axes. At this stage, it is impossible to discern from a split-second video of the mass whether it is spinning about the y axis, the z axis, or engaging in some other complex motion. However, the motion in the past and the future is meaningless since we are only interested in the present moment as it relates to angular momentum.

We can see that, for instance, the I_{yzy} item in I explains how much the angular velocity's 3 component contributes to the angular momentum's L2 component. The $I_{yz} = I_{zy}$ entry in I also reveals how much the 2 component of the angular velocity contributes to the L3 component of the angular momentum owing to the symmetry of I . In the first scenario, we can see that this is only the right component of the velocity times the distance from the y axis if we group the product of the several values as $(y)z$. With $(2z)y$, it is the reverse grouping in the later instance. However, there is one component of y and one factor of z in each instance, which is why I is symmetrical.

Ordinary motions

How can we manage space's overall motion? In other words, what happens if an item is simultaneously rotating and translating? Since the different mass components in motion aren't revolving around the origin as they were before we can't write $v = r$. By writing the motion as

the sum of a translation and a rotation, we can use to compute L (relative to the origin) and the kinetic energy T . Any point on the body may be used as the point when using the theorem P in the formula.

DISCUSSION

We'll show that the only situation in which we can extract anything valuable is when P is the object's CM. After that, the theorem states that the motion of the body is the result of adding the CM's motion to its rotation. Let the body instantly spin around the CM with z angular velocity (i.e., with respect to the frame whose origin is the CM and whose axes are parallel to the fixed-frame axes) while the CM moves with velocity V . Let $R = (X, Y, Z)$ represent the location of the CM in relation to the origin, and let $r = (X, Y, Z)$ represent the location of a given mass in relation to the CM. The location of a mass component with respect to the origin is thus given by $r = R + r$. Let v be the velocity of a mass component with respect to the CM ($v = \dot{r}$). The velocity in relation to the origin is thus given by $v = V + v$.

The theory of parallel axes

Think about the unique situation when the CM spins around the origin at the same angular speed as the body rotates around the CM or $V = R$. For instance, you may do this by piercing the body with the base of a rigid "T" and rotating both the T and the body around the (fixed) line of the "upper" section of the T (the origin must pass through this line). After that, we are in the pleasant position where the body's points move in fixed circles around the axis of rotation. This arises mathematically from the formula $v = V + v = R + r = r$.

Primary axes

Although the lengthy phrases in the preceding part may have seemed a little disturbing, it turns out that most of the time we can get by without them. The utilization of the major axes of a body, which we shall explain below, is the method for avoiding all of the aforementioned chaos. The inertia tensor I often includes nine nonzero elements, six of which are independent since I is symmetric. The x , y , and z variables in the integrals in I rely naturally on the coordinate system they are measured with respect to; the inertia tensor, however, also relies on the set of orthonormal basis vectors selected for the coordinate system. Any orthonormal collection of basis vectors is acceptable given a blob of material and an arbitrary origin, but there is one unique set that greatly improves the quality of all our computations. The major axes are the name given to these unique basis vectors. They may be characterized in many identical ways, including: The orthonormal basis vectors for which I is diagonal, or for which are the major axes. The primary axis of many items are readily apparent. Take a uniform rectangle in the x - y plane as an example. The x and y axes should be parallel to the sidewalls, with the origin chosen to be the CM.

Although it is often used, the origin need not be the CM. Each origin is accompanied by a set of main axes. The one-index object I_1 appears more like a component of a vector than a matrix, thus technically we should be putting I_{11} or I_{xx} instead of I_1 , etc., in this matrix. However, using two indices becomes difficult, so we'll be careless and simply use I_1 , etc. Because all the off-diagonal components in the inertia tensor disappear by symmetry, the major axes are unmistakably the x , y , and z axes. For instance, since there are equivalent points (x, y) for each point (x, y) in the rectangle, meaning that the contributions to xy_{dm} cancel in pairs. Additionally, since $z = 0$, any integrals involving z are also zero [7], [8].

Axis where $I = I$ is known as a primary axis. In other words, a primary axis is a particular direction that has the feature that if points along it, then L does, too. The orthonormal set of three vectors with the properties 1, 2, and 3 are thus the major axes of an object. Because the vectors 1, 2, and 3 are just $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ in the frame in which they are the basis vectors, the three assertions in Eq. (9.23) are equal. Think of a spinning object that has a constant angular speed around a fixed axis. If no torque is required, this axis becomes a main axis. Therefore, the item is "happy" to spin about a primary axis in a certain sense. By definition, a set of primary axes is a collection of three orthonormal axes, each of which has this characteristic. For the following reason, this definition of a primary axis is equal to the prior definition. Assume that the object revolves around a fixed axis 1 for which $L = I \dot{\theta} \hat{1} = I \dot{\theta} \hat{1}$. Next, we see that L is fixed since $\hat{1}$ is expected to be fixed. Consequently, $dL/dt = 0$. In contrast, we assert that L points along axis 1 (i.e., $L = I \dot{\theta} \hat{1}$) if the object is rotating along a fixed axis 1 and if $dL/dt = 0$.

This is accurate because, in the event that L does not point along $\hat{1}$, picture placing a dot on the item someplace along L . The dot will eventually have revolved around the fixed vector $\hat{1}$ after a brief interval. However, because we may have rotated our axes around $\hat{1}$ and begun the operation at a later time (this argument assumes that $\hat{1}$ is fixed), the line of L must always pass through the dot. As a result, we see that L has changed, defying the presumption that $dL/dt = 0$. So, L must in fact point in the direction of $\hat{1}$. The absence of any torque required for a rotation around a principal axis means that if an object is pivoted at the origin and if the origin is the only location to which any force is applied (implying that there is no torque surrounding it), the object can rotate with a constant angular velocity. It won't work if you attempt to put up this scenario with a non-principal axis.

A basic idea in physics, motion may be divided into several categories depending on how it moves. One such kind of motion that combines translational and rotational motion is general motion. We must take into account both angular and linear variables, as well as their mathematical connections, in order to correctly represent universal motion.

Motion Translation

The easiest kind of motion to understand is translational motion. It entails the non-rotational movement of an item from one location to another. Displacement, velocity, and acceleration are important parameters related to translational motion. The act of moving an item from one location in space to another is known as displacement. It has both a direction and a magnitude since it is a vector quantity.

Acceleration:

The pace at which a person's speed changes in relation to time is called acceleration. It is likewise a vector quantity, and its definition is the time-dependent derivative of velocity:

Motion Rotational

On the other hand, rotational motion entails the spinning or rotating of an item around a fixed axis. Angular displacement, angular velocity, and angular acceleration are important properties of rotational motion.

The term "angular displacement" denotes a shift in an object's angular position or orientation. It is often measured in radians and is comparable to displacement in translational motion. Linear displacement and angular displacement are related by:

Motion that combines rotation and translation

When an item experiences both translational and rotational motion, general motion results. It may be difficult to mathematically express this combination, yet it is necessary to comprehend how diverse real-world things move. Both angular and linear variables must be taken into account in order to completely explain general motion. Let's use a rolling wheel as an example to demonstrate this idea.

Think of a wheel moving over a level surface. It moves both in a translational and a rotational direction as it rolls [9], [10].

Longitudinal Displacement

The change in the wheel's surface location is shown by the symbol

Axisymmetric Displacement

This denotes a shift in the wheel's orientation or rotational angle.

Linear Speed

The pace at which the wheel's location moves along the surface is known as its linear velocity. Metres per second (m/s) is the unit of measurement.

Angular Speed

The wheel's rotational speed is expressed as its angular velocity. Radians per second (rad/s) is the unit of measurement.

The linear acceleration

A wheel's linear acceleration is a measure of how quickly its linear velocity changes. It is expressed in square metres per second (m/s^2).

Acceleration at an Angle

The wheel's angular acceleration is a measure of how quickly its angular velocity changes. Radians per second squared (rad/s^2) is the unit of measurement. We may use the following equations to explain how these parameters are related to one another:

General Motion's Equation of Motion

We may combine rotational and translational kinematic equations to model generic motion. For instance, the linear displacement equation of motion. These equations show the connection between angular and linear motion in general motion.

Applications of General Motion in Real Life

In many different disciplines and businesses, general motion theory is essential because it enables us to analyse and build systems with both translational and rotational components. Here are a few real-world uses for generic motion:

Automotive Engineering:

When designing and analysing automobiles, engineers must take into account both the rotation of the wheels and the vehicle's linear velocity down the road. Acceleration, braking, and steering are just a few examples of how general motion concepts are employed to enhance vehicle performance.

Robotics:

Wheels, joints, and manipulators on robots often allow for both translational and rotational mobility. Programming and operating robotic systems need a solid understanding of general motion principles.

Aerospace Engineering:

Translational motion in the air or space and rotational motion for stability and control make up the complicated motion experienced by aircraft and spacecraft. Designing aircraft systems requires a thorough understanding of general motion.

Sports science:

Athletes often combine angular and linear motion in their actions. It is necessary to comprehend general motion concepts in order to analyse and improve their performance.

Asymmetric free top

The free symmetric top is a prime example of using Euler's equations in practice. Think of an object with the CM as the origin and two of its major moments being equal. Assume the item is remote from any external forces in space. Although it is not required, a square cross section, for instance, would result in two equal moments, we will pick the object to have cylindrical symmetry around some axis. The symmetry axis and any two orthogonal axes in the cross-section plane via the CM are thus the major axes. Let's choose the x_3 axis as the symmetry axis. The second moments then become $I_1 = I_2$ and I_3 . First, we'll examine the situation from the perspective of a person standing still on the body, and then we'll examine it from the perspective of a person standing motionless in an inertial frame. Although the mathematics involved in this issue aren't too difficult, it may be challenging to understand what all the different vectors are doing, as is the case with most topological problems. Additionally, due to the noninertial frame of reference in the body-frame analysis that follows, intuition is made much more challenging. Let's check it out nevertheless [11], [12].

CONCLUSION

Consider the instantaneous rotation of a rigid body around an axis. This may alter over time, but for the time being, all that matters is how it appears at any particular moment states that the angular momentum is provided by $\mathbf{L} = \mathbf{I} \boldsymbol{\omega}$, where $\mathbf{I}$ is the inertia tensor determined with regard to a specific origin and a specific set of axes (and is, of course, expressed in the same basis). Using the primary axes (relative to the selected origin) as the basis vectors of our coordinate system makes everything, as usual, much prettier. These axes will rotate in relation to the fixed reference frame since they are fixed in relation to the spinning object. $\mathbf{L}$ adopts the lovely shape on this base where the components of along the major axes are 1, 2, and 3. To put it another way, the components are obtained by projecting the vector $\mathbf{L}$ in space onto the instantaneous primary axes. On the one hand, we may put $\mathbf{L}$ in the attractive form by writing it in terms of the rotating primary axes. On the other hand, since the primary axes themselves are changing, expressing $\mathbf{L}$ in this manner makes it difficult to predict how it will vary over time. The advantages end up being more than the disadvantages, thus we will always utilize the primary axes as our basis vectors.

REFERENCES:

- [1] J. S. Siegel *et al.*, “Statistical improvements in functional magnetic resonance imaging analyses produced by censoring high-motion data points,” *Hum. Brain Mapp.*, 2014.
- [2] L. M. Hamm, J. Black, S. Dai, and B. Thompson, “Global processing in amblyopia: A review,” *Front. Psychol.*, 2014.
- [3] L. F. Cuturi and P. R. Macneilage, “Optic flow induces nonvisual self-motion aftereffects,” *Curr. Biol.*, 2014.
- [4] G. Parigi *et al.*, “Long-range correlated dynamics in intrinsically disordered proteins,” *J. Am. Chem. Soc.*, 2014.
- [5] F. E. Udawadia and T. Wanichanon, “Control of uncertain nonlinear multibody mechanical systems,” *J. Appl. Mech. Trans. ASME*, 2014.
- [6] J. Bhattacharya, S. Bhattacharyya, S. Minwalla, and A. Yarom, “A theory of first order dissipative superfluid dynamics,” *J. High Energy Phys.*, 2014.
- [7] S. Rezaeian, Y. Bozorgnia, I. M. Idriss, N. A. Abrahamson, K. W. Campbell, and W. J. Silva, “Damping scaling factors for vertical elastic response spectra for shallow crustal earthquakes in active tectonic regions,” *Earthq. Spectra*, 2014.
- [8] B. Yousefi and C. K. Loo, “Development of biological movement recognition by interaction between active basis model and fuzzy optical flow division,” *Sci. World J.*, 2014.
- [9] M. H. Giga, Y. Giga, and N. Požár, “Periodic total variation flow of non-divergence type in $\mathbb{R}^n$,” *J. des Math. Pures Appl.*, 2014.
- [10] M. A. Kumakhov, “Peculiarities of charged particle motion in an annular channel with electrified walls,” *Tech. Phys.*, 2014.
- [11] M. J. Mienaltowski and D. E. Birk, “Structure, physiology, and biochemistry of collagens,” *Adv. Exp. Med. Biol.*, 2014.
- [12] P. Schopp, H. Graf, M. Maurer, M. Romanovas, L. Klingbeil, and Y. Manoli, “Observing relative motion with three accelerometer triads,” *IEEE Trans. Instrum. Meas.*, 2014.

CHAPTER 10

CONSERVATION OF ANGULAR MOMENTUM

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ABSTRACT:

A basic tenet of physics known as the conservation of angular momentum is essential for explaining how spinning systems and objects behave. According to this rule, unless an external torque acts on an isolated system, its total angular momentum stays constant. The idea of angular momentum conservation, its mathematical derivation, and its importance in numerous physical processes are all covered in this abstract. We also go into the wider ramifications of this concept for comprehending the dynamics of our cosmos as well as its useful applications. A basic tenet of physics known as the conservation of angular momentum is essential to comprehending how rotating objects behave. The idea of angular momentum conservation is examined in this abstract along with its relevance to a number of scientific fields, such as quantum mechanics, astrophysics, and classical mechanics. The cross product of the position vector and linear momentum of an item yields the vector quantity known as angular momentum. The rule of conservation, which asserts that in the absence of external torques, the total angular momentum of a closed system stays constant, is one of the fundamental ideas regulating angular momentum. This conservation rule has broad repercussions that affect a variety of physical phenomena. The behavior of spinning tops, celestial bodies, and the operation of gyroscopes are all explained in classical mechanics by the conservation of angular momentum. It also serves as the foundation for understanding how angular momentum is conserved in collisions, where the total of angular momenta before to a collision equals the amount after, offering important insights into how such occurrences turn out.

KEYWORDS

Angular momentum, Conservation, Dynamics, Energy, Velocity.

INTRODUCTION

A puck of mass m moving on frictionless ice is connected to a skinny vertical pole of radius R by a horizontal rope that is 4 lengths long. At first, the puck moves at speed v_0 in what is effectively a circle around the pole. The puck is dragged into the string as it encircles the pole and finally collides with it. What property of the motion is conserved? What speed does the puck have just before it collides with the pole? A horizontal thread that travels through a small hole in the frictionless table is tied to a block of mass M that is sliding on the surface of the table. The block begins moves at speed v_0 in a circle around the hole that has a radius of 4. What amount remains constant as you gently draw the thread through the hole? What is the block's speed in relation to the hole at a distance of r ?

The possible effectiveness

Find the $V(r)$ that, given L , leads to a spiral route with the formula $r = r_0 k$. Make E equal to zero. Advice: To apply Eq. (7.9), first find an expression for r that has no's in it.

Kepler's laws and gravity

A circle's orbit, create Kepler's third rule from scratch for a circular orbit by utilizing $F = ma$.

Falling towards the sun

Consider the scenario in which the earth is tragically and abruptly halted in its orbit before being permitted to plummet radially into the sun. When will this be finished? Assume that the initial orbit is roughly circular using the data from Appendix J. Tip: Visualise the radial route as a portion of a very small ellipse [1], [2].

Orbits that intersect

Around their CM, two masses, m and $2m$, orbit. The orbits do not overlap if they are circular. However, they do if they are very elliptical. What is the eccentricity's lowest value at which they cross?

Impact parameter

Demonstrate that the impact parameter is equal to the distance b specified in try this:

- (a) Geometrically, by demonstrating that the dotted line is b distance from the origin.
- (b) Analytically, by establishing that the b equals br by having the particle approach from infinity with speed v_0 and impact parameter br .

Closest approach

A planet of mass M is far away when a particle with speed v_0 and an impact parameter of b is launched.

- (a) Determine the distance of closest approach to the planet starting from scratch
- (b) Establish that the hyperbola discussion's findings, which state that the planet's closest approach distance is $k/(+1)$, are consistent with your response to section

Skimming a planet

On the gravitational field of a planet, a particle moves on a parabolic orbit and skims the surface during its closest approach. There is mass density on the globe. What is the angular velocity of the particle as it skims the surface with respect to the planet's centre?

Parabola

A planet of mass M circles a mass m body in a parabolic orbit using the formula $y = x^2/(44)$ and a focal length of 4. There are three approaches to determine the angular momentum:

- (a) Determine the closest approach speed.

Consider the location $(x, x^2/44)$, where x is a sizable value. At this stage, find approximate expressions for the speed and impact parameters.

Circle to parabola

A spacecraft circles a planet in a circular path. It gives a rapid push, causing an f -fold increase in speed. What should f be if the force points in the direction where the objective is to convert the orbit from a circle to a parabola? the direction of tangency? If the force is in a different direction, does your response change? If the push is pointing in a radial direction, how far is it from the object?

DISCUSSION

Determine the lengths of the semi-major and semi-minor axes, assuming that defines an ellipse for 0 to 1, and demonstrate that the outcomes are consistent.

Potential for repulsiveness

Consider the "anti-gravitational" potential $V(r) = \gamma/r$, where $\gamma > 0$, or more often, the electrostatic potential between two similar charges. What fundamental modification has been made to the analysis. Establish the lack of existence of the circular, elliptical, and parabolic orbits. For the hyperbolic orbit, create a diagram similar to that. The total L for a group of particles is just the sum of the L s of all the particles. The vector $\mathbf{r} \times \mathbf{p}$ has several interesting features, making it an excellent object to study. One of these is the conservation rule discussed in Theorem 7.1, which gave us the opportunity to introduce the concept of "effective potential" in Section 7.2. Later on in this chapter, we will also introduce the idea of torque, which is represented by the fundamental equation $\tau = dL/dt$ (corresponding to Newton's $\mathbf{F} = d\mathbf{p}/dt$ law) [3], [4].

In the world, angular momentum issues may be classified into two categories. We will see that it is necessary to find how L varies with time since, as we will see, the answer to any rotating issue always boils down to utilising $\tau = dL/dt$. And since L is a vector, it may vary due to (1) a change in length or (2) a change in direction (or a combination of both effects). In other words, if we write $L = L\hat{\mathbf{L}}$, where $\hat{\mathbf{L}}$ is the unit vector in the L direction, then L may vary depending on whether L or $\hat{\mathbf{L}}$ changes, or both.

The first of these examples, that of constant L , is the most straightforward. Think of the genesis as the centre of a rotating record. Every term in the sum has the condition that the vector $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ is perpendicular to the record. The record will accelerate in a precise manner that we will quickly decide whether we apply a tangential force in the right direction. Nothing strange is occurring in this situation. The record moves more quickly if we apply pressure. L continues to point in the same general direction, but now with a bigger magnitude. In fact, we may fully disregard the notion that L is a vector in this kind of situation. Everything will be OK if we can simply handle its magnitude L . The focus of this chapter is on this first example.

The second instance, however, when L changes direction, may become pretty confusing. The topic of the next chapter will be whirling tops and other items that spin, which have a propensity to make people's heads spin as well. The whole idea in these instances is that L is truly vector. And in order to understand what is happening, we really need to see the situation in three dimensions, unlike in the constant- L scenario.¹ The straightforward equation in Eq. (8.1) provides the value of the angular momentum of a point mass. But we need to understand how to compute the angular momentum of an extended object in order to deal with setups in the real world, which inevitably include numerous particles. The task of doing this. Only rotations about the z axis, or an axis parallel to the z axis, will be covered in this chapter.

Pancake object in the x-y plane

Consider a flat, rigid body moving arbitrarily in the x - y plane (both translating and spinning); What is the body's angular momentum with relation to the coordinate system's origin?² If we consider the body to be made up of m_i mass particles, then the angular momentum of the whole body is equal to the total of the m_i mass particles' individual angular momenta, or $L_i = \mathbf{r}_i \times \mathbf{p}_i$. The entire angular momentum is hence $\mathbf{L} = \sum \mathbf{L}_i$. Instead of a sum, we would have an integral for

a continuous distribution of mass. The masses' positions and momenta affect L . The momenta themselves depend on how quickly the body is rotating and translating. Finding the relationship between L and the distribution and mobility of the component masses is our objective in this case. We shall demonstrate how the outcome will specifically concern the body's geometry [5], [6].

Only things that move in the x - y plane and resemble pancakes will be discussed in this section. In addition to determining an equation for the kinetic energy, we will compute L in relation to the origin. Non-pancake items are covered in Section 8.2. Note that the vector $L = r \times p$ always points in the direction of z since the r and p of all the masses in our pancake-like objects always reside in the x - y plane. This aspect, as previously indicated, is what makes these pancake scenarios simple to handle. L varies exclusively in proportion to its length, not in any other way. So the equation dL/dt will have a simple form when we finally come to it. Before we examine universal motion in the x - y plane, let's first examine a specific situation. The distinction between these two situations and the one between the two fundamental $F = dp/dt$ cases is basically the same. The vector p may fluctuate due to its fluctuating magnitude, in which case $F = ma$ holds true (assumes m is constant). Alternatively, p could alter due to a change in direction, in which case $F = mv^2/r$, the centripetal-acceleration assertion, applies. (Alternatively, a mixture of these consequences could occur.) The first scenario looks a little more logical than the second. Keep in mind that since L contains the vector r , it is defined relative to a specified origin. Therefore, it is useless to inquire what L is without mentioning the origin we have picked [7], [8].

The Non-planar Objects

We limited the subject to pancake objects in the x - y plane. However, almost all of the conclusions we reached apply to nonplanar objects, given that the rotation's axis is parallel to the z axis and that we are only interested in L_z and not L_x or L_y . So let's exclude the pancake hypothesis and proceed with the outcomes we found above. Consider an object that is initially spinning around the z axis. Allow for z -direction extension of the object provide the L_z for each pancake when the object is divided into pancakes parallel to the x - y plane. Additionally, we may deduce that the L_z of the whole object is equal to the total of the L_z of all the pancakes since the L_z of the entire object is equal to the sum of the L_z of all the pancakes. It doesn't matter if the pancakes' z values vary. As a result, for every object rotating around the z axis, we obtain $I_z = \int (x^2 + y^2) dm$ and $L_z = I_z \omega$, where the integration spans the body's full volume we'll figure out the I_z for a variety of nonplanar objects. The analysis in this chapter is not completely generic even provides the L_z for any item because (1) we are constraining the axis of rotation to be the (fixed) z axis and even with this constraint, an object outside the x - y plane could still be considered x and y components of L that are nonzero, but we only discovered the z component. This second observation is odd yet accurate we'll go into great depth about it [9], [10].

Because we can calculate the overall T by summing the T 's of all the pancake slices, continues to provide the T for a non-planar object rotating along the z axis also apply for a non-planar object when the CM is translating and the object is rotating around it (or, more accurately, along an axis that is parallel to the z axis and passes through the CM). In reality, these two equations remain true regardless of which way the CM's velocity V points on this chapter, however, we'll assume that all velocities are on the x - y plane.

Force

We shall now demonstrate that, under certain circumstances (described below), the torque, which we refer to as, is equal to the rate of change of angular momentum. Therefore, $\tau = dL/dt$. This is the rotating equivalent of the linear momentum equation $F = dp/dt$. The fundamental concept is simple, but there are two nuanced problems. One deals with the forces that operate within a group of particles. The second is concerned with the origin's potential acceleration (the origin is the location from which torque and angular momentum are derived). To keep things organised, we'll address three examples that become progressively more complex as we demonstrate the basic conclusion. We may apply the outcome from this chapter's derivation of $\tau = dL/dt$ for entirely generic motion in the next chapter. If the axis of rotation is parallel to the z axis, you may provide a more precise proof of the statement that $\tau = dL/dt$. However, because the general proof isn't much more challenging, we'll deliver it in this chapter and put an end to it. Understanding the behaviour of things and the underlying forces that control the world depends critically on the basic idea of force in physics. Forces are present everywhere and shape the dynamics of the physical world, whether it is the push of a car's engine, the pull of gravity on a falling item, or the tension in a stretched rubber band. The notion of force will be thoroughly examined in this in-depth examination, and its origins, kinds, measurements, effects, and application to daily life will all be covered [11], [12].

The History of the Concept of Force

The early civilizations are where the idea of force first appeared. Early people started to gain an intuitive grasp of force via their observations and interactions with the world. This idea was often described in terms of struggle and opposition. The way that civilizations understood force changed along with them.

Perspectives from the past

Greece in the past: Aristotle, a Greek philosopher, made important advances to our knowledge of force. He distinguished between natural and unnatural motion, the former requiring no power and the latter requiring it. For instance, it was thought that a stone falling to the ground had a natural desire to remain at rest, therefore its motion was deemed natural.

Middle Ages:

During this time period, ideas of force merged with those of philosophy and religion. According to the dominant theory, which was in keeping with the Aristotelian viewpoint, an outside force was necessary to maintain an item in motion.

Galileo Galilei and the Renaissance:

The study of force underwent a paradigm shift during the Renaissance. Galileo Galilei disproved the conventional wisdom by performing experiments that demonstrated that force was not required to keep objects in motion. His contributions established the groundwork for a more scientific perspective on force.

The Definition of Force in Modern Physics

A more accurate and quantitative definition of force has been supplied by modern physics. Newton's equations of motion are used in classical mechanics, which deals with macroscopic objects moving at regular speeds, to define force.

First Law of Motion of Newton

The first law of motion, sometimes known as the law of inertia, asserts that, absent an external force, an object at rest will stay at rest and an object in motion will continue to travel in a straight path at a constant speed. This rule emphasises the idea that an object must be subjected to force in order to alter its state of motion.

Second Law of Motion by Newton

Force, mass, and acceleration are related by Newton's second law. According to this formula, $F = ma$, the force exerted on an item is equal to the object's mass times its acceleration. This formula expresses how much force is required to alter an object's motion.

The Third Law of Motion by Newton

According to Newton's third law, there is an equal and opposite response to every action. This law emphasises how forces in pairs interact with one another. When one thing pulls on another, the second object pulls back with a force that is equal to and opposing the first object's. This concept serves as the foundation for understanding how forces interact in pairs and is shown in a variety of real-world situations.

Different Forces

There are many different sorts of forces, and each has distinct properties. Understanding these kinds of pressures and how they affect things in different ways is crucial.

a. Normal Force:

A contact force that operates perpendicular to the surface that an item is in touch with is known as a normal force. It stops items from passing through solid surfaces and opposes the pull of gravity.

b. Frictional Force:

Friction is a force that counteracts the propensity of two surfaces in contact to move relative to one another. It is essential for daily tasks like walking, driving, and playing sports.

c. Tension Force:

When a rope, cable, or other flexible connection is pulled tight, tension is the force that is communicated through it. It is necessary for many mechanical systems and buildings, including lifts and bridges.

Non-Contact Forces:

The attraction between two masses is the gravitational force. It is a basic force in the universe that drives the motion of all mass-containing objects, including planets, moons, and asteroids.

The electromagnetic force, which combines magnetic and electric forces. It controls how charged particles behave, how electronics work, and how magnets interact with one another.

Atomic interactions are caused by interactions between the strong and weak nuclear forces. Protons and neutrons are held together by the strong force, whereas radioactive decay is governed by the weak force.

Force Measurement

In engineering and research, it is crucial to measure force precisely. Force is measured using a variety of devices and methods, many of which are based on basic physics concepts. Hooke's law, which asserts that a spring's force is exactly proportional to its displacement from its equilibrium position, is used in spring scales. These scales are often used in daily applications to measure forces. A mechanical force is transduced into an electrical signal using load cells, which are transducers. They are used in industrial settings for machine load weighing and monitoring.

Strain Gauges:

Strain gauges calculate the amount of material deformation caused by applied force. The applied force is calculated from variations in resistance brought on by strain.

Balance Scales: Balance scales quantify force via the equilibrium principle. An equivalent force provided to a counterweight balances the gravitational pull imposed on an item.

Modern force sensors often use piezoelectric materials or other cutting-edge technology to detect force precisely.

The Force's Effects

Objects are affected by forces in a variety of ways, including as changes in motion, deformation, and the transfer of energy. From engineering to astrophysics, an understanding of these impacts is essential.

Changing Conditions

Acceleration:

When an object accelerates, it signifies that its velocity has changed. The direction of the force exerted determines the direction of acceleration.

Deceleration:

When a force resists an object's motion, it experiences negative acceleration, also known as deceleration, and finally comes to a halt.

Circular Motion:

A force may create circular motion when it operates perpendicular to the object's velocity. This may be seen in natural phenomena like as planetary orbits and rotating objects.

Elastic Deformation:

When a force is applied, certain materials, such as rubber bands and springs, deform elastically. When the force is withdrawn, they assume their former shape.

Plastic Deformation:

When other materials, like metals, are exposed to stresses over a particular threshold, plastic deformation occurs. When the force is withdrawn, they do not revert to their former shape.

Transmission of Energy

a. Work:

A force does work when it moves an item across a distance. The amount of energy that a force has transmitted is measured as work.

b. Potential and Kinetic Energy:

Forces also alter an object's potential and kinetic energy. For instance, when an object travels vertically, gravitational forces may change its potential energy, and forces can affect the kinetic energy of an item by altering its speed or direction.

CONCLUSION

There are several applications of the conservation of angular momentum in physics and engineering. This theory offers a basic understanding of how rotating objects and systems, such as celestial bodies and spinning tops, behave. We can anticipate and understand a broad variety of events, including the precession of gyroscopes, the motion of the planets in our solar system, and the merger of massive stars into black holes, using the mathematical formulation of angular momentum conservation. Additionally, the conservation of angular momentum has real-world applications in a number of disciplines, including astronomy, sports, and aerospace engineering. This idea is used by engineers to create stable satellite orbits, and by astronomers to investigate the rotational dynamics of galaxies. Angular momentum conservation is used by athletes to do elegant spins and flips in sports like gymnastics and figure skating. In conclusion, the conservation of angular momentum is a fundamental idea in both classical mechanics and contemporary physics. It is an essential tool for understanding the behaviour of spinning systems on Earth and in the universe due to its concise mathematical formulation and wide application. Angular momentum conservation will surely continue to be a guiding concept in our quest to understand the intricate dynamics of the physical world as we dive further into the secrets of the cosmos.

REFERENCES:

- [1] K. Y. Bliokh, J. Dressel, and F. Nori, "Conservation of the spin and orbital angular momenta in electromagnetism," *New J. Phys.*, 2014.
- [2] C. R. Greenshields, R. L. Stamps, S. Franke-Arnold, and S. M. Barnett, "Is the angular momentum of an electron conserved in a uniform magnetic field?," *Phys. Rev. Lett.*, 2014.
- [3] F. Bouchard, H. Mand, M. Mirhosseini, E. Karimi, and R. W. Boyd, "Achromatic orbital angular momentum generator," *New J. Phys.*, 2014.
- [4] E. Pisanty, S. Sukiasyan, and M. Ivanov, "Spin conservation in high-order-harmonic generation using bicircular fields," *Phys. Rev. A - At. Mol. Opt. Phys.*, 2014.
- [5] R. Cheng and Q. Niu, "Dynamics of antiferromagnets driven by spin current," *Phys. Rev. B - Condens. Matter Mater. Phys.*, 2014.
- [6] A. Fleischer, O. Kfir, T. Diskin, P. Sidorenko, and O. Cohen, "Spin angular momentum and tunable polarization in high-harmonic generation," *Nat. Photonics*, 2014.
- [7] M. Monteiro, C. Cabeza, and A. C. Marti, "Rotational energy in a physical pendulum," *Phys. Teach.*, 2014.

- [8] E. Vasiliev, “Rates of capture of stars by supermassive black holes in non-spherical galactic nuclei,” *Class. Quantum Gravity*, 2014.
- [9] N. Sakai *et al.*, “A chemical view of protostellar-disk formation in L1527,” *Astrophys. J. Lett.*, 2014.
- [10] A. D. Gilbert, X. Riedinger, and J. Thuburn, “On the form of the viscous term for two dimensional Navier-Stokes flows,” *Q. J. Mech. Appl. Math.*, 2014.
- [11] M. D. Weinberg, “Direct simulation Monte Carlo for astrophysical flows - II. Ram-pressure dynamics,” *Mon. Not. R. Astron. Soc.*, 2014.
- [12] N. Fang, “Difficult concepts in engineering dynamics: Students’ perceptions and educational implications,” *Int. J. Eng. Educ.*, 2014.

CHAPTER 11

THE CONCEPT OF ENERGY AND MOMENTUM

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ABSTRACT:

Fundamental physics ideas like energy and momentum are crucial for understanding how physical systems behave. Momentum is a vector variable that reflects an object's motion, while energy is a scalar number that indicates a system's capacity to do work. The main ideas and connections concerning energy and momentum in classical physics are summarized in this summary. According to the energy conservation principle, the overall energy of an isolated system stays constant throughout time. This rule has significant effects on how physical systems behave, from simple mechanical systems to intricate electromagnetic and thermodynamics processes. It enables us to understand the natural world better and develop technology by enabling us to analyse and forecast how energy is changed and transported within a system. On the other hand, momentum is a function of an object's mass and velocity. If there are no outside forces operating on an isolated system, the law of conservation of momentum states that its overall momentum will stay constant. Understanding the motion of things, from celestial body orbits to particle collisions in particle physics experiments, depends heavily on this idea. The idea of relativistic energy and momentum, which derives from Einstein's theory of special relativity, is also briefly covered in this abstract. Special relativity, in contrast to conventional physics, demonstrates the interconnection and transformation of energy and momentum, resulting to the famous equation $E=mc^2$, which connects energy (E), mass (m), and the speed of light (c).

KEYWORDS:

Energy, Mass, Momentum, Photon, Speed.

INTRODUCTION

The relativistic formulae for the energy and momentum of a heavy particle, $E = mc^2$ and $p = mv$, are derived after accepting the facts that the energy and momentum of a photon are $E = hv$ and $p = h/c$ (where ν is the frequency of the light wave and h is Planck's constant). Consider a mass m that splits into two photons as a hint. Take a look at this decay both in the mass's rest frame and in a frame when the mass has speed v [1], [2].

Photons colliding

There are two photons, each with energy E . They come together at an angle θ , producing a particle of mass M . M , what is it?

Mass Gain

A tiny mass m initially at rest collides with a big mass M travelling at speed V and adheres to it. What is the weight of the final product? Work within the range of M and m .

Two-body decay

Mass MA , which is stationary, decays into mass MB and mass MC . What are MB and MC 's energies? What are the current events?

Threshold energy

An identical stationary particle and a particle of mass m collide. What is the energy threshold for a final state with N mass m particles? (The "threshold energy" is the least amount of energy required for a process to take place.)

Head-on collision

A stationary ball of mass m collides elastically with a ball of mass M and energy E . Show that the mass M 's ultimate energy equals

Although this issue is a bit complicated, you can save a lot of problems by noticing that the equation you find for E_r must have as a root $E_r = E$. (Why?) There are many intriguing constraints you might accept as compensation for slogging through the chaos.

Compton scattering

A stationary electron and a photon come into contact. Show that the resultant wavelength, r , is given in terms of the original wavelength, λ , by $r = \lambda + \frac{h}{m c} (1 - \cos \theta)$, (12.88), where m is the mass of the electron. This is true if the photon scatters at an angle θ (see Fig. 12.14). A photon has an energy of $E = h \nu = hc/\lambda$.

System of masses

Think about a dumbbell with two equal masses, m . With its centre pivoting at the end of a stick, the dumbbell rotates (see Fig. 12.15). The energy of the system is $2 m v^2$ if the mass-movement speed is v .

The system is at rest when seen as a whole. As a result, the system's mass must be $2 m$. (Consider putting it inside of a box so you can't see what's happening inside.) Pushing on the stick (when the dumbbell is in the "transverse" position indicated in the image) and demonstrating that $F dp/dt = Ma$ causes the system to act as if it had a mass of $M = 2 m$ will persuade you that this is really how the system behaves.

Relativistic harmonic oscillator

Under the influence of a force $F = m \omega^2 x$, a particle with mass m travels down the x axis. B is the amplitude. demonstrate that the time is provided by

Relativistic rocket

Take into account the relativistic rocket from Section 12.6. Let's say that mass is transformed into photons in the rocket's rest frame at a rate $\dot{m}$. As a function of v , determine the time t in the ground frame. (Unfortunately, this cannot be inverted to get v as a function of t .) You must analyse a rather challenging integral. Choose your preferred approach: pencil, book, or computer.

Dustpan of relativity

An initial relativistic speed is applied to a dustpan with mass M . It collects dust with mass density (measured in the lab frame) ρ per unit length on the floor. Find the rate (measured in the lab frame) at which the mass of the dustpan-plus-dust-inside system is growing at the speed of

Relativistic dustpan II

Take into account the configuration in Problem 12.11. Find $v(x)$, $v(t)$, and $x(t)$ if the dustpan's starting speed is V . Here, every quantity is measured in relation to the lab frame.

DISCUSSION

Determine the force on the dustpan-plus-dust-inside system (caused by the freshly acquired dust particles crashing into it) as a function of v in both the dustpan frame and the lab frame, and demonstrate that the values are equivalent in both [3], [4].

Relativistic cart

The speed of a long waggon is relativistic. In the ground frame, sand is dumped into the cart at a rate of $dm/dt = \dot{m}$. Assume you are pushing the cart to maintain it going at a steady pace while standing on the ground near where the sand is dumped. What is the force acting on the ground between your feet? Show that the results are equivalent by computing this force in both the ground frame (your frame) and the cart frame.

Relativistic cart

The speed of a long waggon is relativistic. In the ground frame, sand is dumped into the cart at a rate of $dm/dt = \dot{m}$. Consider pulling on the front of the cart while running with it to maintain a consistent speed of v . How much pressure does your hand exert on the cart? (Assume the cart is constructed from the strongest material feasible.) Show that the results are equivalent by computing this force in both the ground frame and the cart frame (your frame).

Various frames

(a) A thread of length 4 and constant tension T connects two masses m . At the same time, the masses are released, collide, and stay together. What is the resultant blob's mass, M ?

(b) From the perspective of a frame travelling to the left at speed v , think about this situation. From section (a), the energy of the resultant blob must be Mc^2 . Prove that by calculating the work done on the two masses, you arrive at the same outcome.

Splitting mass

One end of a massless thread, T , is tied to a wall, while the other end, M , is attached to a mass. The string's starting length is the mass is let go of. With no initial relative speed, the rear half of the mass separates from the front half halfway to the wall. How long does it take the front half to get to the wall overall?

Relativistic leaking bucket

Let's substitute a massless bucket with a starting mass M of sand for the mass M . The bucket loses sand along the path to the wall at a rate of $dm/dx = M/4$, where m stands for the mass at later points. Take note that both dm and dx are negative in this case.

Accidents and decays

A photon and mass collide with a photon with energy E and a mass m clash. They come together to create one particle. What is the mass of this particle? How fast is it moving? A particle and a photon are produced as a result of the decay of a stationary mass M . What is the particle's mass if its speed is v ? What is the photon's energy? Three photons moving at a speed of v with a mass m . As shown in Fig., it decomposes into three photons, two of which

move at angles of 120 degrees (in the lab frame) and one of which moves forward. How energetic are these three photons?

Perpendicular photon

An energetic photon with mass M collides. M 's mass scatters obliquely. What is the energy of the resultant photon if it goes perpendicular to the direction of the input photon, as seen in Fig. 12.24? A second perpendicular photon * A mass m collides with another mass m at rest while both are travelling at a speed of $4c/5$. Upon contact, a photon with energy E travels in the opposite direction of its initial trajectory, whereas a photon with mass M travels in a different direction according to Fig. 12.25. What is M in terms of E and m ? In terms of m , what is the greatest value of E for which this configuration is feasible transform into photons * A mass m travelling at v mph decomposes into two photons. As shown in Fig. 12.26, one photon goes perpendicular to the initial direction, while the second photon moves off of E at an angle. demonstrating that if $\tan = 1/2$, then v/c equals (ratio It just so happens that the golden M 's inverse is $5/2$).

Maximum mass

A photon and a mass m are moving in the same direction. They come together directly and produce a new particle. How should the photon and mass m be split up if the system's total energy is E , in order to maximise the mass of the resultant particle?

Equal angles

A photon with energy E strikes a mass m that is stationary. What is if, as illustrated the mass m and the subsequent photon (with unknown energy) scatter at identical angles with respect to the original photon direction? What does mc^2 mean in the limit $E \rightarrow 0$, [5], [6].

Higgs production

In a few years, if it exists, the Higgs boson, a hypothetical elementary particle, should be experimentally observed. Combining protons and antiprotons in a high-energy particle accelerator is one method of generating it. How much energy is needed to make the Higgs in the following scenarios given that the rest energy of a proton (and antiproton) is about 1 GeV and the Higgs' rest energy is around 100 GeV

(b) The momenta of a proton and an antiproton are equal and the opposite?

Maximum energy

(a) A particle with mass M breaks down into many particles, some of which might be photons. In the event that one particle has mass m and

What is the most amount of energy that m can have if the total mass of all the other products is? The total 4-momentum of the other products, P , should be written as $P^\mu P_\mu = P^2$, and then squared, to represent the conservation of energy and momentum. The approach from Problem could be helpful.

(b) In beta decay, a neutron breaks down into a proton, an electron, and a neutrino (which, for the time being, is effectively a photon). $E_n = 939.6$ MeV, $E_p = 938.3$ MeV, $E_e = 0.5$ MeV, and $E_\nu = 0$ are the rest energies. What is the electron's maximum allowable energy? A neutrino? Describe your findings.

A collision and force

At first, two identical masses m are separated by x and at rest. One of them is propelled towards the other by a continual force F until they collide and cling together. How much mass does the final particle have?

Pushing on a mass

A mass m begins at rest in (a). You use a steady force F to push against it. How long does it take the mass to go a distance of t ? (In this case, the lab frame is used to measure both t and x .)

(B) Eventually, m 's speed will become close to c . It turns out that it moves close enough to c to stay (about) a constant distance (as measured in the lab frame) behind a photon that was released at $t = 0$ from the beginning location of m for a very long period. How far away is this?

Momentum conundrum

A massless rope with tension T connects two equal masses. According to Fig. 12.28, the masses must travel along parallel lines at a speed of v . The multitudes are then brought together once the restrictions are lifted. They run into one other and merge into a single blob that moves to the right. Is the following logic sound? If the answer is "no," explain why the one or more of the four phrases is/are incorrect.

"The direction of the forces acting on the masses is y . As a result, the masses' momentum in the x direction remains unchanged. However, since they hit with some relative speed, the mass of the resultant blob is more than the total of the original masses. In order to maintain p_x constant, the speed of the resultant blob must be smaller than v , which causes the whole device to slow down in the x direction .

Motion of a rocket

Rocket energy = $M/(1 + v)$ in the ground frame, as was noted towards the conclusion of the first solution to the rocket issue in Section 12.6. Retrieve this conclusion by adding the energy that the photons in the ground frame contain.

Relativistic strings

Two masses A mass m is positioned next to an identical one. A relativistic thread with tension T links them together. A speed of $3c/5$ is abruptly acquired by the front bulk. How far will the masses diverge from the initial point before they collide?

Relativistic bucket

The bucket advances towards the wall at a constant speed which is one of the conclusions. Without utilizing the method of determining the N -limit of several masses, derive this again. Time dilation suggests that since the sand-entering-cart occurrences occur at the same spot in the ground frame, the sand enters the cart in the cart frame more slowly, or at a rate of $/$. The sand enters with velocity v and gradually settles on the cart, losing momentum at a rate of $(/)vv$.

Therefore, this must be the amount of pressure your hand is applying on the cart. The force on your feet would be v in the cart frame, but we discovered earlier that it is $2v$ in the ground frame. If this were the only change in momentum in the situation, then we would have a

problem. This runs counter to the idea that longitudinal forces are constant throughout all frames. What is the answer to this seemingly paradoxical situation? The explanation is that your mass is dropping as you push the trolley. We shall demonstrate that while you are travelling at speed v within the cart frame, mass is constantly being transferred from you to the cart, which is at rest. Revert briefly to the ground frame. As we saw above, the mass of the cart system with the sand inside rises at a constant pace in the ground frame. As a result, in the ground frame, the energy of C grows at a rate. You need to provide the remaining half of this energy as the sand only gives of it. Since you are losing energy at this pace and you are at rest in the ground frame, it follows that you are also losing mass at this rate. This provides the momentum shift we've been lacking. The following is the quantitative justification [7], [8].

Return to the cart frame now. You only lose mass at a rate of $(21/1)/\gamma$ due to temporal dilation. This mass transitions from travelling at speed v (i.e., beside you) to speed zero (i.e., at rest on the cart) in a straight line.

By combining this result with the v result we discovered for the sand, we can conclude that the overall rate of momentum loss is $2v$. According to the math in the ground frame above, this is the force that the ground exerts on your feet [9], [10].

Note that since your speed was 0 while the computation was being done in the ground frame, we did not have to worry about your increasing mass. In light of this, your momentum was always zero, regardless of what happened to your mass [11], [12].

CONCLUSION

Two of the most basic and pervasive ideas in the field of physics are energy and momentum. They provide a framework for comprehending how systems behave at different sizes and have several real-world uses in engineering and technology. The first law of thermodynamics, which states that energy cannot be generated or destroyed but only changed into another form, is known as the conservation of energy. Numerous natural processes, such as the heat flow in a steam engine and the chemical reactions in our bodies, are based on this idea. It has transformed how we might use energy for social good, enabling improvements in transit, power production, and a wide range of other areas. The conservation of momentum, which is governed by Newton's third rule of motion and the law of conservation of momentum, also sheds light on how moving things behave. Understanding how celestial planets behave, how cars work, and how technology is made all depend on this idea. Albert Einstein's theory of special relativity, which was further developed, considerably deepened our knowledge of energy and momentum. It demonstrated that mass itself may be considered of as a type of energy by revealing how closely related energy and mass are via the famous equation $E=mc^2$. This realization has important ramifications for cosmology, nuclear physics, and particle physics energy and momentum are fundamental building blocks of contemporary physics and engineering, not merely abstract ideas. Our knowledge of the cosmos and the forces that propel technological advancements that have an influence on every part of our lives are still being shaped by their conservation principles and linkages. Energy and momentum continue to be crucial compass points for our exploration of the universe's secrets and new scientific and technological frontiers.

REFERENCES:

- [1] S. Pfenninger, A. Hawkes, and J. Keirstead, "Energy systems modeling for twenty-first century energy challenges," *Renewable and Sustainable Energy Reviews*. 2014.

- [2] O. Ellabban, H. Abu-Rub, and F. Blaabjerg, “Renewable energy resources: Current status, future prospects and their enabling technology,” *Renewable and Sustainable Energy Reviews*. 2014.
- [3] G. Seyfang, S. Hielscher, T. Hargreaves, M. Martiskainen, and A. Smith, “A grassroots sustainable energy niche? Reflections on community energy in the UK,” *Environ. Innov. Soc. Transitions*, 2014.
- [4] S. P. S. Badwal, S. S. Giddey, C. Munnings, A. I. Bhatt, and A. F. Hollenkamp, “Emerging electrochemical energy conversion and storage technologies,” *Frontiers in Chemistry*. 2014.
- [5] G. S. Alemán-Nava *et al.*, “Renewable energy research progress in Mexico: A review,” *Renewable and Sustainable Energy Reviews*. 2014.
- [6] A. B. Awan and Z. A. Khan, “Recent progress in renewable energy - Remedy of energy crisis in Pakistan,” *Renewable and Sustainable Energy Reviews*. 2014.
- [7] M. T. Islam, S. A. Shahir, T. M. I. Uddin, and A. Z. A. Saifullah, “Current energy scenario and future prospect of renewable energy in Bangladesh,” *Renewable and Sustainable Energy Reviews*. 2014.
- [8] N. Fumo, “A review on the basics of building energy estimation,” *Renewable and Sustainable Energy Reviews*. 2014.
- [9] E. Burman, D. Mumovic, and J. Kimpian, “Towards measurement and verification of energy performance under the framework of the European directive for energy performance of buildings,” *Energy*, 2014.
- [10] C. Zalengera, R. E. Blanchard, P. C. Eames, A. M. Juma, M. L. Chitawo, and K. T. Gondwe, “Overview of the Malawi energy situation and A PESTLE analysis for sustainable development of renewable energy,” *Renewable and Sustainable Energy Reviews*. 2014.
- [11] I. Kimura, D. Inoue, K. Hirano, and G. Tsujimoto, “The SCFA receptor GPR43 and energy metabolism,” *Frontiers in Endocrinology*. 2014.
- [12] A. Ranjan and L. Hughes, “Energy security and the diversity of energy flows in an energy system,” *Energy*, 2014.

CHAPTER 12

AN OVERVIEW ON VECTORS

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ABSTRACT:

Mathematical constructs known as vectors have applications in a variety of disciplines, including physics, engineering, computer science, and economics. They allow us to describe and examine a variety of occurrences in our environment since they both reflect size and direction. We provide a succinct introduction of vectors, their characteristics, and their importance in several fields in this abstract. We may express and manage quantities with both magnitude and direction using mathematical constructs called vectors. They are essential for explaining forces, motion, and other physical phenomena. Vectors are also crucial in mathematics and have useful uses in a wide range of disciplines, including engineering, computer graphics, economics, and geography. The capacity to perform operations like addition, subtraction, and scalar multiplication is one of the distinctive characteristics of vectors. With the use of these processes, vectors may be used to solve a variety of difficult issues, whether they involve analyzing motion, resolving equations, or optimizing systems.

KEYWORDS:

Engineering, Frame, Physics, Relative speed, Vectors.

INTRODUCTION

Fundamental mathematical constructs known as vectors are pervasive in many disciplines, including computer science, economics, physics, and engineering. In this thorough investigation of vectors, we will look into their historical development, characteristics, uses in many fields, and operations. You will have a thorough knowledge of vectors and their importance in both theoretical mathematics and practical situations by the conclusion of this in-depth examination [1], [2].

Introduction to Vectors

Simplest forms of vectors are mathematical objects that may describe magnitude and direction. They are often represented in two- or three-dimensional space as arrows. Numerous physical quantities, including force and velocity in physics and income and profit in economics, may be represented by vectors. We will examine the definition and representation of vectors [3], [4].

Vector Types

Depending on their characteristics and uses, vectors may be divided into a number of different categories. In this lesson, we'll look at a few different types of vectors, including scalar, unit, and free vectors. Each kind has a particular function in various settings.

Vector Spaces

Fundamental mathematical structures known as vector spaces are made up of vectors and follow a set of rules and characteristics. We will examine the characteristics of vector spaces

and their relevance to linear algebra, including closure under vector addition and scalar multiplication [5], [6].

Addition and Subtraction of Vectors

Basic operations like vector addition and subtraction enable us to mix vectors or determine their differences. We'll investigate the geometric and algebraic approaches to carrying out these operations and comprehend their importance in manipulating vectors.

Multiplication of Scalars

Another basic vector operation is scalar multiplication. It enables us to change the size of vectors while maintaining their orientation. We shall go into scalar multiplication's characteristics and geometric meaning [7], [8].

Dot Product

A vector operation that produces a scalar output is the dot product, sometimes referred to as the scalar product. We'll look at how to compute the dot product, how to interpret it geometrically, and how to use it in physics, engineering, and geometry.

Cross Product

Three-dimensional space only uses the cross product, sometimes referred to as the vector product. We'll look at the cross product's formula, characteristics, and physics uses, notably in defining rotational motion.

DISCUSSION

As combinations of their constituent parts along coordinate axes, vectors in three dimensions may be visualised. We will investigate where to locate these elements and how to switch between several vector representations.

Vector Equations

For explaining relationships between vectors, vector equations are an effective tool. We will go into vector equations, discussing issues like vector addition, dot and cross products, and their uses in engineering and physics [9], [10].

Vector Projections

We may locate a vector component along another by using vector projections. We'll look at how vector projections are computed as well as their importance in fields like physics and computer graphics.

Geometry of vectors

Two-Dimensional Vector Geometry

This part will cover subjects including vector magnitudes, direction angles, and the connection between vectors and geometry as well as two-dimensional vector geometry.

Three-dimensional Vector Geometry

We will investigate vector geometry in three dimensions to further our grasp of two dimensions, covering ideas like direction cosines, vector equations of lines and planes, and the geometric meaning of vector operations.

Engineering and Physics

In physics and engineering, vectors are often employed to describe physical quantities and represent real-world occurrences. We will look at how they are used in fluid dynamics, electromagnetism, and mechanics.

Computer Animation and Graphics

In computer graphics and animation, vector operations are essential because they make it possible to manipulate objects in virtual spaces. We'll look at how 2D and 3D visuals are produced using vectors.

Finance and Economics

Vectors are used in economics and finance to express economic indicators and examine market patterns. We'll look at how vectors are used to economic system modelling and financial forecasting.

Geographical Information Systems (GIS)

To effectively describe geographic data, GIS depends on vectors. We will explore the mapping, navigation, and spatial analysis applications of vectors in GIS.

Calculus of vectors

To explain more complicated processes, vector calculus blends vector operations with conventional calculus. Concepts including gradient, divergence, curl, and line integrals will be covered.

Tensors

Vectors and matrices are multidimensional mathematical objects known as tensors. We will discuss the idea of tensors and how they are used in engineering and science.

Quantum Mechanics

In the mathematical formulation of quantum physics, vectors are essential. We shall briefly discuss the usage of vectors in the description of quantum states and operators.

Physical laws' format

The idea that all inertial frames are equal is one of Special Relativity's postulates. A physical rule must thus hold true in all frames if it does so in one frame. It would be able to distinguish between frames if not. According to the preceding section, " $f = ma$ " cannot be a physical rule. The assertion cannot be true in all frames since the two sides of the equation change in different ways when we switch between frames. A statement may only include 4 vectors if it has any possibility of being true in all frames. Suppose " $A = B$ " is a 4-vector equation that holds true in frame S . Then, if we apply a Lorentz transformation (designated M) from one frame S to another frame S' to this equation, we obtain therefore, frame S likewise conforms to the legislation. Naturally, many 4-vector equations (such $F = P$ or $2P = 3P$, for example) are false in any frame. Only a limited subset of these equations, such $F = mA$, hold true in all frames and hence in at least one frame.

Scalar equations, such $P \cdot P = m^2$, are another way that physical laws may be expressed. By definition, a scalar is a quantity that is independent of the frame (as we have shown the inner product to be). A scalar assertion is valid in all inertial frames if it is true in one of them.

Higher-rank "tensor" equations, such those that occur in electromagnetism and general relativity, may also represent physical laws. Tensors are something that may be thought of as objects built up from 4-vectors, but we won't get into them in this article. Special instances of tensors include scalars and four-vectors.

This is all quite similar to what is going on in three dimensions. $F = ma$ is a potential law in Newtonian mechanics since both sides are 3-vectors. It depends on whatever axis you designate as the x axis, but $f = m(2a_x, a_y, a_z)$ is not a viable law since the right-hand side is not a 3-vector. If you choose the eastward direction as your x direction and a particle accelerates in that direction, the force is $2ma$ eastward.

However, if you choose eastward as your y direction, the force will be ma eastward. A law cannot provide two distinct outputs depending on your arbitrary selection of axis labels. The assertion that a particular stick is 2 metres long is an example of a frame-independent statement (under rotations). This is okay since the norm, which is a scalar, is involved. Nevertheless, if you claim that the stick has an x component of 1.7 metres, then this is not true in every frame. I've added some strict selections, God told his cosmic commanders. One is the requirement that your physical rules be expressed in terms of four vectors.

Velocity addition

B and C move to the right and to the left, respectively, in A's frame, with speeds u and v , respectively. What is the speed difference between B and C? In other words, the velocity-addition formula is derived using 4-vectors.

Relative speed

In the lab frame, two particles travel along the routes shown in Fig. with a speed of v . The angle between the trajectories is 2° . What is the apparent speed of one particle to the other? Yet another relative speed. In the lab frame, particles A and B travel along the routes in Fig. with speeds u and v , respectively.

The angle between the trajectories what is the apparent speed of one particle to the other? Linear motion acceleration. A spacecraft begins at rest in relation to frame S and accelerates properly at a constant rate where is the spaceship's proper time gives the spaceship's speed with respect to S. Assume that the spacecraft has a 4-velocity of V and a 4-acceleration of A . Regarding the appropriate moment: (a) Using the formula explicitly, locate V and A in frame S. In the spaceship's frame, note V and A . Confirm that between the two frames, V and V transform like 4-vectors.

A key idea in both physics and daily life, relative speed describes how things move in relation to one another in a dynamic, linked universe. In many disciplines, from physics and engineering to transportation, sports, and even interpersonal relationships, an understanding of relative speed is essential. We will examine the origins, mathematical formulations, practical applications, and the central role that relative speed plays in our comprehension of motion and interaction in this thorough investigation.

Defining Relative Speed

In essence, relative speed is the pace at which the relative positions of two objects change with respect to a certain frame of reference. The concept of reference frames is introduced in this section, along with a definition of relative speed that is both precise and useful for comprehending motion in relation to an observer. With contributions from ancient thinkers

like Aristotle and ground-breaking discoveries by Galileo Galilei and Sir Isaac Newton, the idea of relative speed has developed throughout the years. We will discuss significant turning points in our knowledge of relative speed throughout history. It is crucial to understand how reference frames affect relative speed. We will look at the idea of invariance, which paves the way for the theory of special relativity by demonstrating how the rules of physics hold true regardless of the reference frame used.

Relative Velocity

A key mathematical idea when discussing relative speed is relative velocity. We will examine relative velocity computation and talk about its importance in physics and real-world problems including vehicle accidents and riverboat issues.

Vector Representation

Vectors may be used to describe relative speed, enabling us to take into account both magnitude and direction. The relevance of vector mathematics in resolving practical issues is emphasized as this subject investigates vector addition and subtraction to discover relative velocities.

Relative Speed in One, Two, and Three Dimensions

One-dimensional motion is not the only use of relative speed. In order to better comprehend the intricacy of relative motion in several dimensions, we will apply our mathematical knowledge to two- and three-dimensional situations.

Transportation and Navigation

Whether speaking about cars, planes, or spaceships, relative speed is crucial in the transportation industry. We will investigate the fundamental importance of relative speed computations in navigation, collision avoidance, and space missions.

Athletics and Sports

Relative speed calculations are crucial in competitive sports including racing, basketball, and football. We'll look at how athletes utilise relative speed to make split-second choices that affect the results of competitions.

Mechanics and Engineering

Understanding relative speed is essential for developing equipment, analysing mechanical systems, and guaranteeing safety in engineering and mechanics. We shall look at its uses in pulleys and gear systems in mechanical engineering.

Celestial mechanics and astronomy

A key idea in celestial mechanics is relative speed, which aids in our comprehension of the movements of the planets, moons, and other celestial entities in the cosmos. We shall examine the mathematical foundations of relative velocities in the universe.

Special Relativity According to Einstein

Our concept of relative speed and motion has been fundamentally changed by Einstein's special theory of relativity. We will look at how special relativity challenged our traditional ideas of relative motion by introducing ideas like time dilation and length contraction.

Relativistic Relative Speed

Relative speed works differently at high speeds compared to classical physics. We will look at how special relativity affects computations of relative speed in near-light-speed settings, as well as the relativistic addition of velocities.

Relative Speed in Human Interaction

Relational speed pertains to interpersonal interactions as well as movements of objects. We'll look at how differences in relative speed impact interactions, perceptions, and relationships among people in diverse social circumstances.

Perspectives from Culture and Society

The way that time and relative speed are perceived varies between cultures and communities. We'll look at how social institutions and cultural norms may affect how quickly life moves forward and how people interact.

The Twin Paradox

A popular thought experiment in special relativity that clarifies the idea of time dilation is the twin paradox. We will analyse this paradox and see how it affects how we think about relative speed and time.

Zeno's Irrationalities

The motion and infinite divisibility paradoxes of Zeno cast doubt on our perception of relative speed. We will examine these contradictions and how they might be resolved in order to get fundamental understanding of the nature of motion [11], [12].

CONCLUSION

A basic idea known as vectors serves as the foundation for a wide range of disciplines and applications. We have studied the characteristics, functions, and applications of vectors during this investigation. A mathematical foundation for expressing quantities with both magnitude and direction is provided by vectors. They are inescapable in characterising a vast array of physical phenomena, from the motion of celestial planets to the forces acting on buildings, due to their dual nature. Scalar multiplication, addition, and subtraction are examples of vector operations that provide us strong tools for tackling challenging issues. Vectors provide an organised and elegant method for a variety of tasks, including projectile trajectory calculation, computer graphic design, resource allocation optimisation, and economic system modelling. Vectors are profoundly ingrained into the structure of our universe and are not only relegated to the field of mathematics. They help us to move around, communicate, and invent. Vectors are a unifying language that connects theory and practical application in everything from engineering wonders to simulations in video games, from the study of market trends to the comprehension of motion in sports. As a result, vectors are more than just impersonal mathematical constructs; they represent the central motif of our comprehension of the physical universe and the means by which we might innovate and overcome the problems of the present. Vectors will remain a crucial tool for understanding the complicated, interrelated world in which we live as science, technology, and daily life advance.

REFERENCES:

- [1] T. Ura, K. Okuda, and M. Shimada, “Developments in viral vector-based vaccines,” *Vaccines*. 2014.
- [2] L. L. Coffey, A. B. Failloux, and S. C. Weaver, “Chikungunya virus–vector interactions,” *Viruses*. 2014.
- [3] R. Bellini, H. Zeller, and W. Van Bortel, “A review of the vector management methods to prevent and control outbreaks of West Nile virus infection and the challenge for Europe,” *Parasites and Vectors*. 2014.
- [4] A. J. da Silva, T. C. Zangirolami, M. T. M. Novo-Mansur, R. de Campos Giordano, and E. A. L. Martins, “Live bacterial vaccine vectors: An overview,” *Brazilian Journal of Microbiology*. 2014.
- [5] S. M. Dizaj, S. Jafari, and A. Y. Khosroushahi, “A sight on the current nanoparticle-based gene delivery vectors,” *Nanoscale Research Letters*. 2014.
- [6] H. Yin, R. L. Kanasty, A. A. Eltoukhy, A. J. Vegas, J. R. Dorkin, and D. G. Anderson, “Non-viral vectors for gene-based therapy,” *Nature Reviews Genetics*. 2014.
- [7] N. Nafissi, S. Alqawlaq, E. A. Lee, M. Foldvari, P. A. Spagnuolo, and R. A. Slavcev, “DNA ministrings: Highly safe and effective gene delivery vectors,” *Mol. Ther. - Nucleic Acids*, 2014.
- [8] D. H. Barouch and L. J. Picker, “Novel vaccine vectors for HIV.1,” *Nature Reviews Microbiology*. 2014.
- [9] X. Peng, C. Zou, Y. Qiao, and Q. Peng, “Action recognition with stacked fisher vectors,” in *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 2014.
- [10] J. D. Suerth, V. Labenski, and A. Schambach, “Alpharetroviral vectors: From a cancer-causing agent to a useful tool for human gene therapy,” *Viruses*. 2014.
- [11] M. R. Liou, Y. W. Huang, C. C. Hu, N. S. Lin, and Y. H. Hsu, “A dual gene-silencing vector system for monocot and dicot plants,” *Plant Biotechnol. J.*, 2014.
- [12] A. Plattner and F. J. Simons, “Spatiospectral concentration of vector fields on a sphere,” *Appl. Comput. Harmon. Anal.*, 2014.